STRENGTH OF MATERIALS

A TEXT BOOK FOR TECHNICAL AND INDUSTRIAL SCHOOLS

BY

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PREFACE

This text is the result of over twelve years' experience in teaching the subject of strength of materials to the students at Pratt Institute. An attempt has been made to present the fundamental and underlying principles with a minimum of mathematics. These principles are applied by the introduction of a large number of examples which are worked out in detail. It is only by the working of original problems that the student is able to test his knowledge of the subject.

Many of the applications are made more particularly to mechanical lines of work, and yet the student who proposes to follow structural work will find the text of great help to him.

The text covers the application of strength of materials in the proportioning of beams, columns, shafting, riveted joints, and in problems dealing with simple stresses.

Chapter I has been made of an introductory nature, the desire being to present very briefly, but concisely, the simple laws of equilibrium, so that the student may be able quickly and accurately to analyze the forces acting in any machine or machine part.

A chapter is inserted dealing with the testing, and the production of the more common materials of construction.

It has been the experience of the author that only by the assigning of original problems in class is the instructor able to test the grasp of the subject the student is securing,

and, therefore, recommends the use of 3×5 cards on which are placed problems which the student has not seen prior to entering class.

The author desires to extend his thanks to his associates, Mr. Frank O. Price, for the chapter on Reinforced Concrete, and Mr. E. H. MacCoul, for checking the problems and valuable suggestions relating to the text.

JOHN P. KOTTCAMP.

BROOKLYN, JUNE, 1919.

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STRENGTH OF MATERIALS

CHAPTER I

CONDITIONS OF EQUILIBRIUM

ART. I. DEFINITIONS

Gravity is the force by which all bodies are attracted toward the earth's center. The force with which gravity attracts any body toward the earth is called the weight of the body.

Mass is the quantity of material in a body.

Force is that which produces or tends to produce motion or change of motion of bodies. It manifests itself to the feeling by a tension or pull, and by a compression or push.

The **Unit of Force** is the weight of a mass of one pound or simply the pound.

A *force* is completely defined when its magnitude, direction, and point of application are known. In Fig. 1 the line

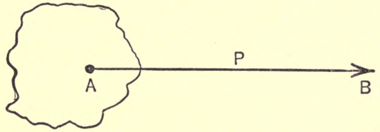


FIG. 1.

AB represents the magnitude of the force *P*; the arrow located at the end of the line *AB* shows that the force is

acting from A toward B, and A indicates the point of application of the force.

The *component* of a force is the effort exerted by that force in any given direction in the plane of the force. In Fig. 2 let the line OB represent to a given scale the force P. From the point B drop a perpendicular BM to the horizontal line OX, and a perpendicular BN from the point B to the line OY. Then the line OM will represent, to scale, the horizontal component of the force P, and the line ON the vertical component of the force P.

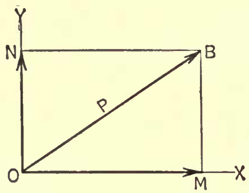


FIG. 2.

ART. 2. MOMENTS

The *moment* of a force is the tendency of the force to produce rotation of a body about any given point. The *moment arm* is the perpendicular distance from the given point to the line of action of the force.

The *measure of a moment* is the product of the force in pounds times the moment arm in feet, hence moments are expressed in foot-pounds. Moments may be expressed in inch-pounds by using the force in pounds and the moment arm in inches. In Fig. 3, let AB represent a force of P pounds, tending to rotate the body D about the point O. Draw OC perpendicular to AB. Then OC, or p, is the moment arm, and the moment equals $AB \times OC = Pp$.

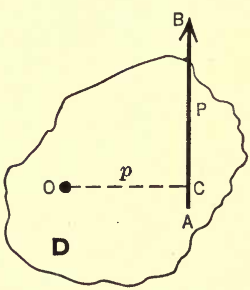


FIG. 3.

In this text moments tending to produce clockwise rotation will be considered positive, and those tending to

produce counter-clockwise rotation will be considered negative.

Prob. 1. A force of 12 lbs. acts at a distance of 20 ins. from a given point. Find the value of the moment of the force; (*a*) in inch-pounds; (*b*) in foot-pounds.

Prob. 2. The pull on a belt is 200 lbs. The radius of the pulley is 24 ins. Find the moment of the belt pull on the shaft.

Prob. 3. Two forces of 50 and 75 lbs. act at distances of 3 ft. and 5 ft. from a given point. If the 50-lb. force exerts a positive moment and the force of 75 lbs. a negative moment, find the resulting moment.

Art. 3. Resultant Force

Any single force which produces the same effect on a body as the combined action of two or more forces is called the resultant of those forces. Take, for example, two men pulling on a vertical hoist. Assume each man exerts a pull of 50 lbs. Their combined effort is 100 lbs. A single pull of 100 lbs. would then have the same effect as the combined pull of the two men. In all cases where the forces are parallel and in the same direction the resultant is easily found by adding together the single forces. When the forces make an angle with each other, the process of finding the resultant force is not so simple.

The resultant of a system of forces can be found either graphically or algebraically. The meaning of resultant force may be made obvious by an easily performed experiment. Secure two small pulleys, a few pieces of stout cord, and several small weights, and arrange the apparatus as shown in Fig. 4. Suppose, for example, the weight at the point A is 5 lbs., at point B 3 lbs., and at the point C 6 lbs. Place a heavy piece of cardboard back of the cords and draw a line parallel to the line CA, making it proportional to the weight of 5 lbs., according to some assumed scale; in like

manner draw another line parallel to the cord CB and make it proportional to 3 lbs. to the same scale. Construct the parallelogram $CFDEC$ by drawing DE parallel to CF and DF parallel to CE. Measure the line CD and, if the experiment has been carefully performed, it will be found that the line CD will measure 6 lbs. to the given scale, and it will further be found that the line CD is vertical. Therefore if the weights W_1 and W_3 were replaced by a single vertical force acting upward at the point C and equal to 6 lbs., this

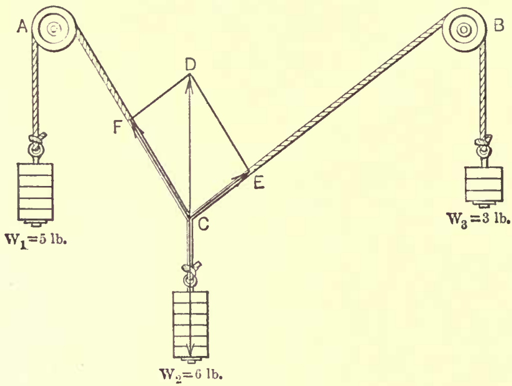


FIG. 4.

force would keep the weight of 6 lbs. from falling. It is to be noted that the resultant force is acting upward. The force of 6 lbs. acting downward is called the *equilibriant*, or the force producing a state of rest.

By varying the weights and changing the lengths of the cords (so as to secure different angles) the student can find the resultant for many other combinations, and he will always find it to be represented by the diagonal of the parallelogram formed by the two forces. From these simple experiments the following law can be stated:

If two forces acting at a point on a given body be represented

in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by the diagonal of the parallelogram passing through this point.

Another truth demonstrated by the above experiment is that the resultant force lies in the same plane with the forces to which it is equivalent. To find the resultant force by a graphical method requires great care in the drawing of all lines and angles, and even then a certain degree of error is introduced.

Another method of determining the resultant of two forces (and this method can be applied where there are more than two forces) is to resolve the forces into their horizontal and vertical components, and then find the algebraic sum of all the horizontal components, indicated by ΣH, and the algebraic sum of all the vertical components, indicated by ΣV. Thus all the forces are replaced by a single vertical force, and a single horizontal force and the resultant can be found from equation,

$$R=\sqrt{\overline{\Sigma H}^2+\overline{\Sigma V}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

PROB. 4. Two forces of 80 lbs. and 120 lbs. make an angle of 30° with each other. Determine the resultant by each of the three methods discussed.

PROB. 5. The thrust on the connecting rod of an engine is 8000 lbs. Find the horizontal and vertical components when the connecting rod makes an angle of 20° with the horizontal, assuming length of connecting rod equals five times the length of the crank.

PROB. 6. Four forces act on a given body at the point O. The first force is horizontal and equals 50 lbs. The second force makes an angle of 60° with the horizontal and equals 60 lbs. The third force makes an angle of 30° with the horizontal and equals 40 lbs. The fourth force makes 120° with the horizontal and equals 45 lbs. Find the magnitude and direction of the resultant.

Art. 4. Conditions of Equilibrium

When any number of forces which are either concurrent or non-concurrent and lie in the same plane tend to produce a state of equilibrium, it has been shown that certain general conditions must be satisfied by the forces. The first of these conditions is that there *can be no resultant force.* The second of the conditions is that there may be no resultant horizontal or vertical component, and for this to be true the algebraic sum of all the horizontal and the vertical components must equal zero. If the forces acting on the body pass through a common point, the above conditions are sufficient to insure a state of equilibrium; but if the forces are non-concurrent, then the third condition, that the algebraic sum of the moments of all the forces about a given point must equal zero, is also necessary to determine completely the state of equilibrium. A system of forces which is in equilibrium may produce either a state of rest or a state of uniform motion of the body on which the system acts. Hence the forces acting on a body which is in a state of uniform motion may be in equilibrium just as well as forces which act on a body at rest.

In the design of a machine it is essential that the forces acting on the various parts of the machine be known, and that these forces be in equilibrium. To analyze these forces carefully it is usual to consider each part of the machine removed and its place taken by a force represented in magnitude and direction by a straight line. When the parts are thus replaced by the forces the machine is spoken of as a "free body." For a complete analysis of the conditions of equilibrium the student is referred to books on elementary mechanics. For the purpose of this work the following general statements will serve:

For equilibrium to exist when,

CASE I. Two forces act on a body:

The forces must be equal in magnitude, opposite in direction and have a common point of application.

CASE II. Three forces act on a body:

(*a*) The forces must lie in the same plane.

(*b*) The lines of action of the forces pass through a common point.

(*c*) The three forces may be represented in magnitude and direction by the sides of a triangle taken in order.

(*d*) If the three forces are parallel, the resultant force equals zero, and the algebraic sum of the moments of the forces about any given point equal zero.

CASE III. Any number of forces act on a body,

(*a*) If forces are concurrent and lie in the same plane, they can be represented in direction and magnitude by the sides of a polygon taken in order.

(*b*) Any one force is equal and opposite to the resultant of all the other forces.

(*c*) The algebraic sum of the horizontal and the vertical components must equal zero.

(*d*) The algebraic sum of the moments of all forces about any given point must be zero.

(*e*) If the forces are parallel, the rule (*d*) under Case III may be applied.

It will be noted that in Case I the forces are necessarily concurrent. In Case II the forces may be either concurrent or parallel, and in Case III the forces may be concurrent, parallel, or non-concurrent, and *in all the cases the forces lie in the same plane.*

PROB. 7. A boiler stay-bolt makes an angle of 25° with the shell. If the steam pressure is 100 lbs. per sq. in. and the stay supports an area 6×9 ins. find the pull acting in the stay-bolt.

PROB. 8. In the locomotive crane shown in Fig. 5 the total load acting at the point *A*, due to the weight of the bucket, coal,

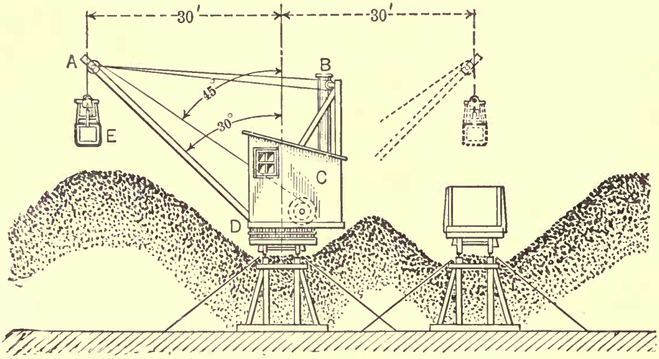


FIG. 5.

and boom is 4000 lbs. The pull in the rope *AC* is 3500 lbs. Find the force tending to crush boom *AD*.

PROB. 9. In Fig. 6 let the weight $W = 1000$ lbs. If the angle $a = 30°$ find the tension in the rod *AC*.

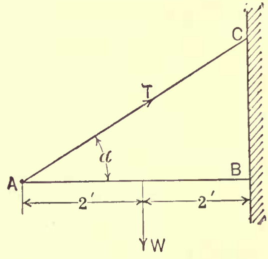


FIG. 6.

CHAPTER II

SIMPLE STRESSES

Art. 5. Unit Stress

The effect of an external force or *load* on a given part of a machine or structure is to set up in that part a resistance which in every case is equal in magnitude and opposite in direction to the given force. This internal resistance is called a *stress*. For example, a rope sustains a load of 1000 lbs., the internal resistance or stress of the rope is, therefore, 1000 lbs.

A *unit stress* is the resistance offered by 1 sq. in. of the material. This unit stress may be expressed in pounds per square inch (written lbs. per $\square''$), or pounds per square foot. Thus, if a bar of steel 2 ins. in diameter sustains a pull of 10,000 lbs., the unit stress will equal 10,000 divided by the area of the bar, or $\frac{10,000}{3.14} = 3188$ lbs. per sq. in.

Stresses may be *direct* or *indirect*. The direct stresses are tension, compression, and shear, and are referred to as tensile, compressive, or shearing stresses.

The external loads or forces producing these stresses are referred to as tensile, compressive, and shearing. The effect of a tensile load is to produce a lengthening or elongation of the part, resulting in a reduction of the cross-sectional area. The effect of a compressive load is to produce a shortening of the part, or an increase in the cross-sectional area,

while the effect of a shearing load is to produce a cutting of the parts.

The indirect stresses are bending or flexure, and torsion or twisting.

In direct stresses the internal resistance is assumed to be uniform on every square inch of the material.

Let P = the external load in pounds;
A = the cross-sectional area in square inches;
S = the unit stress in pounds per square inch.

Then, $$P = AS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) is to be used when the area and the unit stress are known.

$$A = \frac{P}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) is to be used when the load and unit stress are known.

$$S = \frac{P}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) is to be used when the load and area are known.

Example: A round steel bar sustains a load of 12,000 lbs. What should be its diameter if the unit stress is not to exceed 6000 lbs. per sq. in.? Here the unknown quantity is the area, therefore, Equation (3) is to be used, hence

$$A = \frac{P}{S} = \frac{12000}{6000} = 2 \text{ sq. ins.}$$

The area of a circle is given by the equation $A = .7854\ d^2$, where d = the diameter, or

$$d = \sqrt{\frac{A}{.7854}}.$$

The diameter of the steel bar

$$=\sqrt{\frac{2}{.7854}}=\sqrt{2.56}=1.6 \text{ ins.}$$

PROB. 10. A boiler stay-bolt is $1\frac{1}{8}$ ins. in diameter. If the unit stress is not to exceed 7000 lbs. per sq. in., find the load that can be placed on the bolt.

PROB. 11. A wrought-iron tension member in a bridge sustains a load of 60,000 lbs. The cross-section of the member is $2\frac{1}{4}\times4\frac{1}{2}$ ins. Find the unit tensile stress.

PROB. 12. A generator weighing 1800 lbs. is suspended by an eye-bolt $\frac{3}{4}$ in. in diameter. Find the unit stress in the shank of the bolt. (NOTE.—Use the area at root of the thread.)

ART. 6. ELASTIC LIMIT.

Whenever an external load is applied, causing an internal stress, there always results some kind of deformation to the part under load. For example, a weight attached to the end of a rope causes it to stretch, but when the weight is removed the rope will return to its original length. If the load be gradually increased, a point will finally be reached at which, upon removal of the load, it will be found that the rope has been permanently increased in length. This same principle can be applied to a bar of steel under either a tensile or compressive stress. Hence it is found by experiment that up to a certain point materials can be externally loaded and upon removal of the load the materials will return to *their original condition*. Beyond this point a *permanent deformation* or *set* is found to remain upon removal of the load. Further, within the limits of this point the amount of deformation is found to be directly proportional to the load. For example, if a load of 1000 lbs. cause a bar to stretch .001 of an inch, a load of 2000 lbs. may cause it to stretch .002 of an inch The point at which the deformation no longer remains pro-

portional to the load is called the *elastic limit* of the material. It therefore follows that if any part of a machine be loaded beyond its elastic limit and the load be removed, a permanent change of shape of the part will result. Hence in all problems in design no unit stress is to be used which exceeds the elastic limit of the material. Again, it is found that the same material varies in its physical properties, so that tests on different samples of the same material may show a variation in the elastic limit. The *elastic limit* is expressed in pounds per square inch. Table I gives the elastic limit of the more common materials used in engineering construction. It must be understood that these are average values only. For instance, in the case of steel it is found that the elastic limit varies with the physical properties of the steel.

TABLE I

ELASTIC LIMIT

Material.	Tension.	Compression.
Steel: Soft	30,000	30,000
Mild	35,000	35,000
Hard	55,000	60,000
Wrought Iron	25,000	25,000
Cast Iron	6,000	20,000
Stone	....	2,000
Timber (with grain)	3,200	3,000

PROB. 13. In a tension test on a steel rod 0.8 in. in diameter the total load at the elastic limit was 18,000 lbs. Find the elastic limit in pounds per square inch.

PROB. 14. A stone pier carries a load 150,000 lbs. If the unit stress is one-half the elastic limit, find the dimensions of the stone, assuming its cross-section to be square.

PROB. 15. A soft steel tie rod carries a load of 24,000 lbs. Find its diameter if the unit stress equals one-third the elastic limit.

Art. 7. Ultimate Strength

The maximum load in pounds per square inch required to rupture a specimen in tension or compression is called the *ultimate strength* of the material. In the case of tension this is referred to as the *ultimate tensile strength*, and in the case of compression as the *ultimate compressive strength*. The ultimate strength is determined experimentally (see Art. 11) and is expressed in pounds per square inch. Table II gives the ultimate strength in tension and compression of the various materials.

TABLE II

Ultimate Strength

Material.	Ultimate Strength. Tensile, Lbs. per Sq. In.	Ultimate Strength. Compressive, Lbs. per Sq. In.
Hard Steel	100,000	120,000
Structural Steel	60,000	60,000
Wrought Iron	50,000	50,000
Cast Iron	20,000	90,000
Brass (Cast)	24,000	....
Timber (with Grain)	10,000	8,000
Stone	....	6,000

Needless to state, no unit stress in the design of any machine part dare equal the ultimate strength, for this would mean rupture or failure of the part. Therefore, the actual unit stress must be considerably below the ultimate strength to avoid rupture, and must be below the elastic limit to avoid permanent deformation. A margin of safety must always be allowed, which is usually expressed as a factor. The *factor of safety* is a number representing the ratio of the ultimate strength to the actual unit stress, more

commonly called the *unit working stress.* Expressed as an equation,

$$\text{Factor of safety} = \frac{\text{ultimate strength}}{\text{unit working stress}}. \quad . \quad (5)$$

or

$$\text{unit working stress} = \frac{\text{ultimate strength}}{\text{factor of safety}}. \quad . \quad . \quad (6)$$

However, there might be a factor of safety, as far as strength is concerned, and still have a unit working stress which would cause a permanent deformation. To avoid this another term is introduced, called the *real* factor of safety, which is the ratio of the elastic limit to the unit working stress, or expressed as an equation,

$$\text{Real factor of safety} = \frac{\text{elastic limit}}{\text{unit working stress}}. \quad . \quad (7)$$

To distinguish these the factor of safety based on ultimate strength is called the *apparent* factor of safety.

EXAMPLE. A steel tie rod $1\frac{1}{4}$ ins. in diameter is subjected to a pull of 16,000 lbs. Find the apparent and the real factor of safety.

SOLUTION. The area of a $1\frac{1}{4}$-in. rod = 1.22 sq. in. The unit working stress equals

$$\frac{16000}{1.22} = 13{,}110 \text{ lbs. per sq. in.}$$

The *apparent* factor of safety equals $\frac{65000}{13110} = 5$ (nearly).

The *real* factor of safety equals $\frac{35000}{13110} = 2.67$.

Therefore the bar is safe as to strength and as to permanent deformation.

The selection of the proper factor of safety and the determination of the correct unit working stress are largely established by experience and good judgment in machine design. Table III is inserted for the convenience of the student, but it should be borne in mind that these are only average values, and in many special cases must be modified. The factor of *common sense* must be used in determining unit-working stresses.

TABLE III

FACTORS OF SAFETY

Material.	Steady Stress, Buildings, etc.	Varying Stress, Bridges.	Repeated or Reversed Stresses, Machines.
Steel, Hard	5	8	15
Steel, Structural	4	6	10
Wrought Iron	4	6	10
Cast Iron	6	10	20
Timber	8	10	15
Brick and Stone	15	25	30

PROB. 16. A block of wood 10 ins. square standing on end supports 50,000 lbs. What is the factor of safety?

PROB. 17. A boiler stay-bolt $\frac{7}{8}$ in. in diameter sustains a load of 6000 lbs. Find its factor of safety. (Use root of thread.)

PROB. 18. The mild steel piston rod of a 12×18 in. engine is $2\frac{1}{4}$ ins. in diameter. The maximum steam pressure is 100 lbs. per sq. in. Find the factor of safety when the rod is in tension.

ART. 8. SHEAR

A tensile load produces an elongation of the specimen with a resulting reduction in area. As the load passes the elastic limit the elongation increases at a greater rate, with the result that specimen begins to "neck" down, that is, the area toward the center of the specimen is reducing at a

greater rate than anywhere else. This is referred to as local elongation, and is clearly shown in Fig. 7.

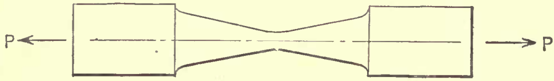


Fig. 7.

The unit working stress in tension is found by dividing the total load by the original area of the specimen. A compressive load causes a shortening of the specimen with a tendency to cause rupture by a splitting or shearing, provided the specimen is not too long, in which case a bending action will result. The usual method of failure in pure compression is shown in Fig. 8. Here again the unit compressive stress equals the total load divided by the original area.

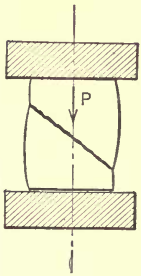


Fig. 8.

A shearing load tends to produce a cutting across. Thus, in the case of a rivet the unit shear would equal the load divided by the cross-section of the rivet.

TABLE IV

Ultimate Shearing Strength

Material.	Ultimate Strength, Pounds per Square Inch.
Timber { With Grain	600
Timber { Across Grain	3,000
Structural Steel	50,000
Wrought Iron	40,000
Cast Iron	20,000
Stone	1,500

Table IV gives the ultimate strength in shear of various materials. There are some cases where the calculation of the unit shear is not so easy as in the case of tension and compression. Consider the case of a bolt where the shank of the bolt is put in tension (see Fig. 9). The unit tensile stress on the shank of the bolt $=\dfrac{P}{.7854d^2}$. The head of the bolt tends to tear away from the shank, setting up a shear between the head of the bolt and the shank. Now assume that the head and shank are two separate pieces and that the

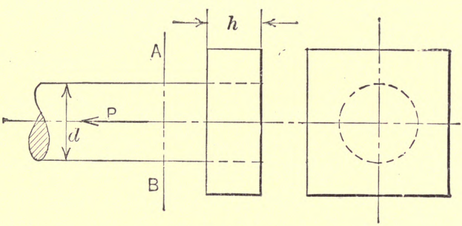


FIG. 9.

shank slips into a hole in the head of the bolt. It is evident that the area of contact between the two will be the circumference of the shank times the length of the head. Hence the shearing area in this case becomes $3.1416 \times d \times h$.

EXAMPLE. Let $P=5000$;

$d=\frac{3}{4}$ in.;

$h=1$ in.

Find the unit tensile stress on the shank of bolt and the unit shearing stress on the head of the bolt.

Let S_t = unit tensile stress;

S_s = unit shearing stress.

$$\text{Then } S_t=\frac{P}{A_t}=\frac{5000}{.7854\times\frac{32}{4}}=\frac{5000}{.442}=11{,}320 \text{ lbs. per sq. in.}$$

$$S_s=\frac{P}{A_s}=\frac{5000}{3.1416\times\frac{3}{4}\times 1}=\frac{5000}{2.356}=2120 \text{ lbs. per sq. in.}$$

PROB. 19. A rivet 1 in. in diameter is subjected to a load of 10,000 lbs. If the rivet is in single shear find the unit shearing stress.

PROB. 20. Assume same rivet as in Prob. 19 to be in double shear. Find the unit shearing stress.

PROB. 21. A wrought-iron bolt $1\frac{1}{2}$ ins. in diameter has a head $1\frac{3}{4}$ ins. long. Find (*a*) the unit stress tending to shear the head and (*b*) the unit tensile stress on the shank of the bolt, when subjected to a pull of 15,000 lbs.

ART. 9. SIMPLE PROBLEMS IN DESIGN

In this article is presented a series of problems involving simple direct stresses. The student is urged in every case to follow some general plan of attack in the solution of these and other problems. In the judgment of the author there are four essential steps in the solution of any problem:

First—**Think.**

Get a clear mental picture of what the problem involves.

Second—**Analyze.**

Figure out exactly what it is that you are trying to find and then determine what elementary principle underlies the problem. A free-hand sketch showing the details of the problem is very helpful in this regard.

Third—**Equate.**

Set down whatever equations are involved. *Do not memorize equations,* but get a firm grip on fundamental principles. *An equation is simply an algebraic expression of a fundamental principle.*

Fourth—**Solve.**

Apply rules of mathematics and make it a point to solve in the simplest manner possible. Keep the unknown quantity on the left-hand side of the equation, and all known factors on the right-hand side.

Get the habit of working problems with the idea of filing them away for reference in the future. Specify clearly what every item in your solution signifies. Check your results and then acquire the feeling of confidence that your answer is correct, and do not rely upon the other fellow's result.

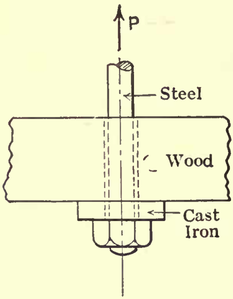


FIG. 10.

PROB. 22. In Fig. 10 the pull on the rod *AB* is 25,000 lbs. Unit working stress of the wood in compression is 800 lbs. per sq. in. Find diameter of the cast-iron washer if the hole in the washer is $2\frac{1}{8}$ ins. diameter.

PROB. 23. Fig. 11 shows a wood test specimen. If the ultimate tensile strength of wood is 10,000 lbs. per sq. in, and the

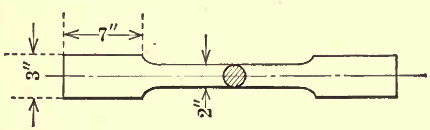


FIG. 11.

ultimate shearing strength is 600 lbs. per sq. in., determine the manner in which the specimen will fail.

PROB. 24. The steam pressure in a boiler is 125 lbs. per sq. in. Each boiler stay supports an area of $7\frac{1}{4} \times 6\frac{3}{8}$ ins. Find the diameter of the stay if the unit stress is not to exceed 5800 lbs. per sq. in.

PROB. 25. A wrought-iron rod $1\frac{1}{4}$ ins. diameter broke under a load of 67,610 lbs. Find the ultimate tensile strength of the wrought iron.

PROB. 26. What pull will be required to break a structural steel rod $2\frac{1}{4}$ ins. in diameter?

PROB. 27. A load of 4000 lbs. is suspended as shown in Fig. 12. Find the diameter of the steel rods A and B, using a factor of safety of 6.

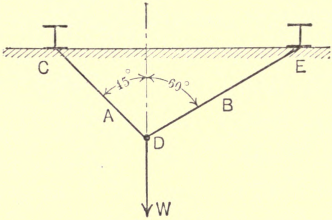


FIG. 12.

PROB. 28. In Fig. 6 assume AB to be a square wooden strut and consider it as under pure compression. Find the size of strut, using a factor of safety of 10.

PROB. 29. If, in Fig. 9, the diameter d of the bolt is, $1\frac{1}{2}$ ins. and the length of head h is 1 in., determine the manner in which the bolt will fail.

PROB. 30. A cast-iron test specimen 1 in. in diameter broke under a load of 75,000 lbs. Find the ultimate compressive strength of cast iron.

CHAPTER III

MATERIALS OF CONSTRUCTION

Art. 10. Testing Machines

The physical properties of the various materials of construction are determined experimentally by means of a testing machine, a common form of which is shown in Fig. 13. By means of this machine tests in tension, compression, shear, and bending can be made. The machine consists of a heavy platen A, mounted upon knife edges B, which in turn are secured to a system of levers connected to a weighing beam C, upon which is mounted the sliding weight D. Power is applied by gearing to the four vertical screws which operate the movable head E. These screws pass through holes in the fixed head, but are independent of the weighing mechanism. Mounted upon the platen are four heavy metal struts F, which support the fixed head H. There is a tapered slot in both the fixed and movable heads. In these slots are inserted tapered jaws for holding the specimen. The machine is provided with several speeds to facilitate operation.

The up-and-down motion of the movable head is controlled by the lever L, which operates a clutch. An open belt runs on the pulley M. When the lever L is thrown to the left the pulley M operates the drive shaft, and when L is thrown to the right the pulley R, on which a crossed belt operates, connects with the drive shaft. The pulley M causes downward motion of the head E, and the pulley R causes upward motion.

When a slow speed is desired the lever L is put in neutral position (directly vertical); the hand-wheel G is turned to the right, which causes the friction gear at P to operate. This results in a very slow rotation of the drive shaft and consequently slow downward motion of the movable head.

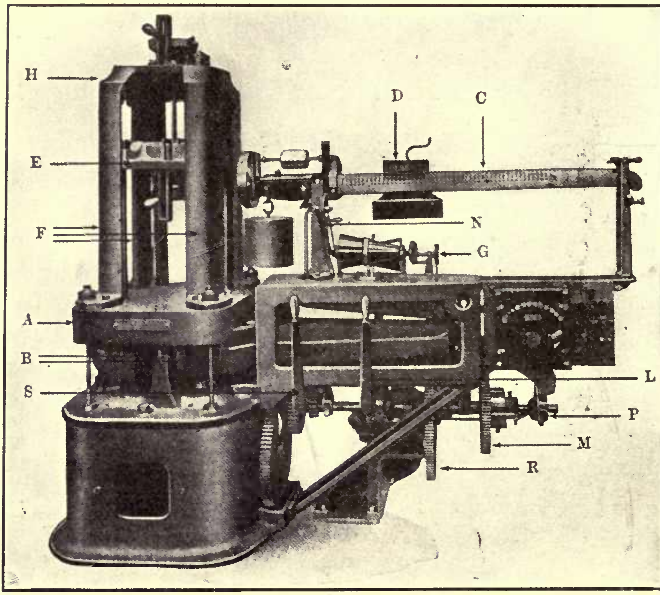


Fig. 13.

The lever S is used to change gears when a quick motion of the head is desired, but this lever must never be operated when the machine is running.

In the case of a tension test the movable head works downward, the specimen being secured in the jaws of both the fixed and the movable heads. This elongates the specimen and transmits the load through the fixed head to the

struts, to the platen, and thence to the weighing mechanism. The poise D is operated by the hand wheel N, so that the load on the specimen can be determined by keeping the weighing beam in balance at all times. The elongation is measured by means of an extensometer which is attached to the speci-

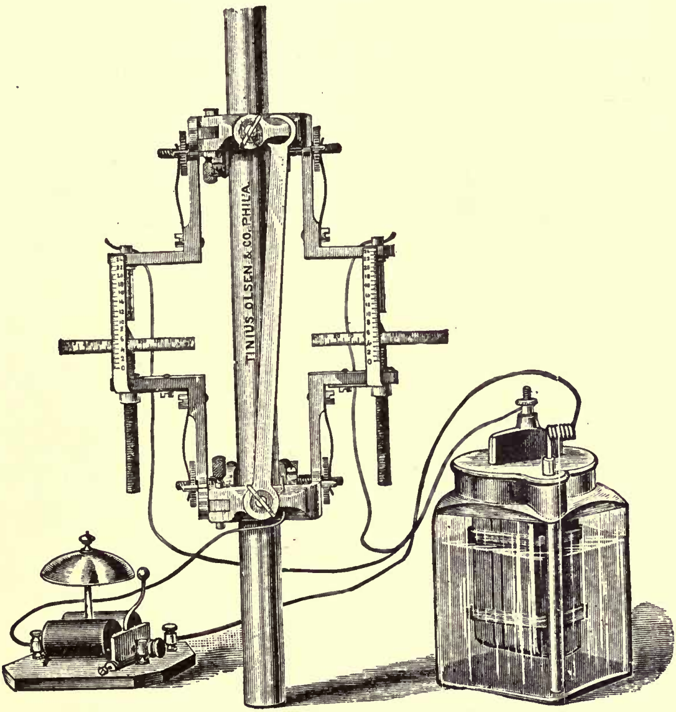


FIG. 14.

men under test. A good form of this instrument is shown in Fig. 14.

In conducting a compression test a flat plate is placed under the movable head and the specimen to be tested is placed between this plate and the platen. Power is applied

and the load on the specimen is transmitted directly to the platen and thence to the weighing mechanism. A counter weight is provided to balance the weight of the levers, platen, etc. Before starting either a tension or compression test it is essential that beam is floating when the riding weight *D* is on the zero mark. In compression tests the deformation is recorded by means of a compressometer, which is constructed somewhat similarly to the extensometer.

The capacity of a machine is expressed in pounds, for example, a 250,000-lb. machine indicates one that is capable of breaking a specimen requiring a load of 250,000 lbs. to produce rupture. These machines are either belt or motor driven. In some cases the machine is provided with an attachment for automatically recording on a chart the load and deformation on the specimen. For more detailed explanation of these machines the student is referred to the trade catalogues.

Art. 11. Tension and Compression Tests

The specimens used for tension tests vary in size and shape with the material to be tested and the purpose for

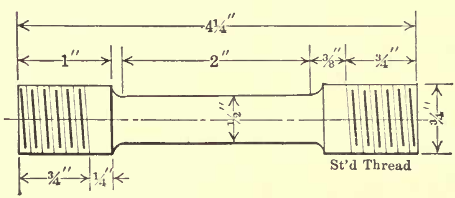


Fig. 15.

which it is intended. Fig. 15 shows a form used for structural steel Fig. 15*a* for cast iron and Fig. 16 a form used for testing boiler plates.

Fig. 17 gives a typical curve, showing the relation between the total load and the total elongation for structural steel.

The five significant results of a tension test are:

(1) The ultimate tensile strength.
(2) The elastic limit.
(3) The per cent elongation.
(4) The per cent reduction of area.
(5) The modulus of elasticity.

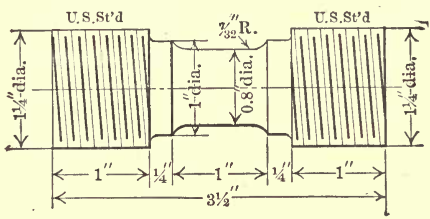


Fig. 15*a*.

The *ultimate strength* is found by dividing the maximum load in pounds required to rupture the specimen by the original area of the specimen in square inches.

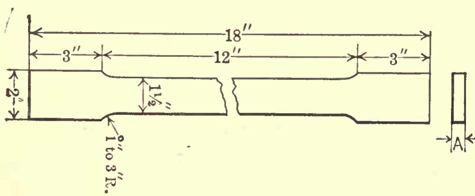


Fig. 16.

The *elastic limit* is found by dividing the load in pounds, at the elastic limit, by the original area of the specimen in square inches.

The *per cent elongation* is determined from the ratio of the total elongation, which is the difference between the final and the original lengths of the part under test, to the original length.

The *per cent reduction in area* is the difference between the original and final areas divided by the original area. In

both the case of elongation and reduction of area the ratio must be multiplied by the factor 100 to express the result in per cent.

The *modulus of elasticity* is the ratio of the unit stress at the elastic limit to the unit deformation at the elastic limit. In the case of a tension test the unit deformation equals the total elongation at the elastic limit divided by the original length of the part under test. If the specimen breaks near either end a correction must be made for the local elongation.

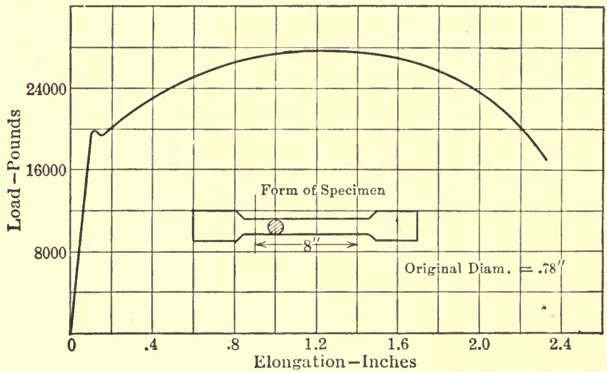


FIG. 17.

Compression tests are conducted in the same manner as tension tests, and the results are figured on the same basis. Fig. 18 gives a typical curve for a compression test of cast iron.

EXAMPLE. Find the five significant results of the tension test shown in Fig. 17, where the original diameter of test specimen was .78 in. and the part under test 8 ins. in length. The final diameter was .553 in. and the final length 10.232 ins.

SOLUTION. From the curve the maximum load is found to be 29,000 lbs. and the load at the elastic limit 19,000 lbs. The total elongation at the elastic limit is 0.0104 in.

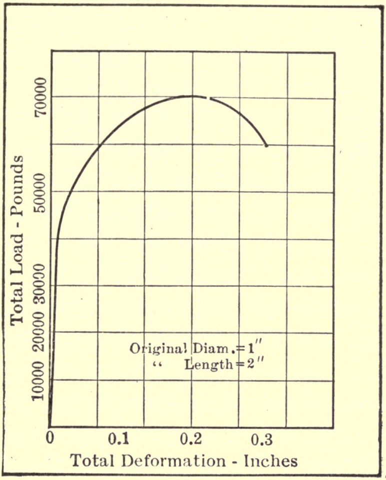


FIG. 18.

Hence:

(1) The ultimate strength $=\dfrac{29000}{.478}=60{,}660$ lbs. per sq. in.

(2) The elastic limit $=\dfrac{19000}{.478}=40{,}000$ lbs. per sq. in.

(3) Per cent elongation $=\dfrac{10.232-8}{8}\times 100=28.$

(4) Per cent reduction of area $=\dfrac{.478-.24}{.478}\times 100=50.$

(5) Modulus of elasticity $=\dfrac{40000}{.0013}=30{,}000{,}000.$

PROB. 31. A 2-in. cube of yellow pine broke under a compressive load of 28,000 lbs. The load at elastic limit was 12,000 lbs. The deformation at the elastic limit was .004 in. Find the ultimate compressive strength, the elastic limit, and the modulus of elasticity.

PROB. 32. A test specimen of boiler plate $2\times\frac{1}{2}$ in. in section broke under a load of 52,000 lbs. The elongation in 8 ins. was 2.2 ins. Find the tensile strength and the per cent elongation.

Art. 12. Cast Iron

Iron, which constitutes the most important element in modern industry, never occurs free in nature, but is always combined with other elements and impurities. The primary source of iron is from the enormous deposits of iron ore in this and other countries. This iron ore is placed, together with a fuel, usually coke, and a flux, usually limestone, in a large vertical, conical-shaped retort called the blast furnace. Under the action of a blast of hot air the charge in the furnace is gradually reduced to a molten state, and the iron, together with other elements, collects in the bottom or hearth of the furnace. At stated intervals the furnace is tapped and the iron conducted either into large ladles, or run directly into molds, producing " pig iron." At a point somewhat above the metal tap is another tap for the removal of the " slag." This slag is a result of the combination of the impurities in the iron ore with the " flux," and is produced in such large quantities that its final disposition is no small problem in the modern iron industry.

If the iron is not to be used directly for the manufacture of steel it is run into molds made of sand. These molds produce a pig weighing about 80 lbs. This pig iron is shipped from the blast furnace to the various foundries, where it is placed, together with a fuel and flux, into a " cupola." Here under the action of a blast of air the pig iron is reduced to a molten state and the impurities again partially removed. From the cupola the iron is poured into ladles and thence into whatever forms of molds that may have been prepared. The student must keep in mind the fact that " pig iron " is the product of the blast furnace and cast iron is the product of the cupola in the iron foundry.

The grade of cast iron can be closely regulated by the cupola charge. Frequently pig iron is charged, together

with better grades of cast iron, into the cupola. In this way the final composition of the cast iron can be regulated.

Cast iron consists of metallic iron with at least 1.5 per cent carbon. It also contains silicon, sulphur, phosphorus, manganese, and other elements. Usually there will be about 90 to 92 per cent of iron and 10 to 8 per cent of other elements in cast iron. The carbon occurs either in the free or " graphitic " form or in the " combined " state. The per cent of carbon materially effects the physical properties of the iron; if the carbon is in the graphite or free state, a gray iron will be the result; if the carbon is in the combined state, a white iron will result.

Silicon in cast iron prevents the combining of the carbon, and a gray iron results. The per cent of silicon varies from .5 to 4 per cent. The silicon tends to prevent blowholes in the castings, thus producing an iron of greater density. Sulphur in cast iron should never exceed .15 per cent; it is considered an undesirable element.

Phosphorus tends to make cast iron more fluid, but also induces great brittleness. For good strong castings phosphorus should not exceed .6 per cent. Manganese tends to produce hardness in the iron.

Cast iron finds its greatest use in machine frames, bed plates, etc., on account of its high compressive strength and the ease with which the iron can be cast into almost any size or shape. Cast iron is not used in tension, owing to its low tensile strength, and to the facts that its per cent elongation is negligible; it has no well-defined elastic limit in tension.

PROB. 33. A cast-iron specimen 1 in. in diameter and 2 in. long broke under a load of 18,000 lbs. Find the ultimate tensile strength of cast iron.

PROB. 34. A cast-iron bearing plate supports a load of 50,000 lbs. Find its cross-sectional area so that the unit compressive stress shall not exceed one-quarter the elastic limit.

Art. 13. Wrought Iron

Wrought iron is defined by Campbell as slag-bearing, malleable iron which does not harden materially when suddenly cooled.

Wrought iron is manufactured by heating pig iron, together with iron ore, scrap iron, and other fettling material in a puddling furnace which is lined or "fettled" with iron oxides in the form of roll scale, high-grade iron ore, or roasted puddle cinder. The furnace is of the reverberatory type, with a capacity of from 300 to 1500 lbs. The fuel used is either gas or bituminous coal, and the flame is allowed to play on the "bath" of metal in the furnace hearth.

The pig iron used for manufacture of wrought iron is known as forge iron, and contains about 1 per cent of silicon, 0.5 per cent manganese, less than 1 per cent of phosphorus, and not over 0.1 per cent of sulphur. The larger the per cent of impurities in the charge the greater the loss of metal during the process.

The melting-down stage requires from thirty to thirty-five minutes, after which there is a stage of from seven to ten minutes during which very high grade iron ore is added, during which time the damper is put on so as to cool the charge and oxidize the impurities before the carbon. This produces the "boil," which lasts from twenty to twenty-five minutes, during which time the puddler stirs the charge with a long iron bar, the slag pouring out into a slag bucket. During the next fifteen to twenty minutes the "balling" period occurs, at which time the puddler divides the bath into a number of balls which are removed from the furnace by means of a pair of tongs suspended from an overhead track. These balls are taken to the "squeezer," where part of the slag is removed by passing the balls through an eccentric vertical roll, leaving the metal in the form of a rectangular

bar. This bar is then put through the rolls, producing what is called "muck" bar, which contains a large percentage of slag and impurities.

The muck bars are placed in a reheating furnace, from which they are taken and again passed through rolls, producing *merchant* bar, or commercial wrought iron. When merchant bar is sheared, piled, and rerolled the resulting material is called *double refined iron.* Coal is generally used as fuel in the puddling furnace. When charcoal is used as fuel and the air forced in through tuyeres a better product is the result. This material is commercially known as charcoal iron.

Wrought iron, owing to its method of manufacture, gives a fibrous fracture. It has a fairly high tensile strength with somewhat less per cent elongation than steel. Wrought iron is easily welded, and for this reason finds its greatest use in parts of machines where shaping by forging is necessary.

The product of the rolling mill is bar iron and plate iron. Owing to the method of manufacture the size of plate is limited. The thickness depends upon the type of service for which the iron is intended.

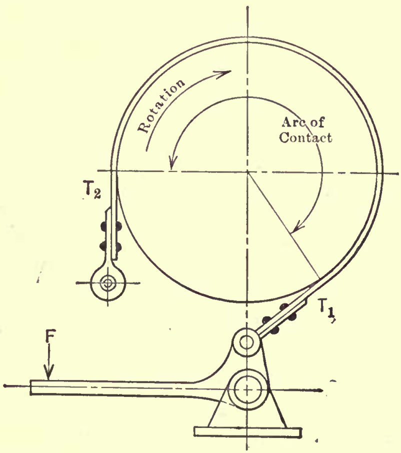


Fig. 19.

Prob. 35. Fig. 19 shows a simple band brake. Maximum tension on the wrought-iron strap is 2000 lbs. If the thickness of the strap is $\frac{3}{16}$ in., find the breadth using a factor of safety of 10.

PROB. 36. Fig. 20 shows a form of malleable-iron chain. The links are 2 ins. wide and $\frac{1}{4}$ in. thick. Find the safe load that the

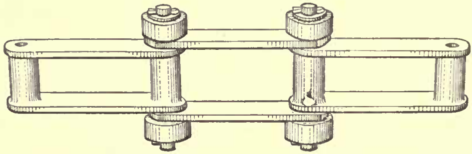


FIG. 20.

chain can carry so that the unit tensile stress shall not exceed 6000 lbs. per sq. in.

ART. 14. STEEL

By the term steel is meant the product of the cementation process or the malleable compounds of iron made in the crucible, the converter, or the open-hearth furnace.

Crucible Steel. Wrought iron and steel scraps, together with charcoal or pig iron, and other ingredients, dependent upon the product desired, are placed in a covered crucible made of about 50 per cent graphite and 50 per cent clay. These crucibles hold between 80 and 100 lbs. of metal, and last from six to eight heats. The crucibles are placed in rows in a regenerative furnace heated by producer gas or coke.

The process requires from four to five hours, at the end of which time the crucibles are withdrawn by a "puller out," who straddles the pit in which the crucibles are placed and draws them out with a pair of tongs. The crucibles are then emptied into a large ladle from which the metal is poured into ingots of any desired shape.

Crucible steel is largely used for the making of instruments and tools for use in high-speed machine work. There are about seventy-five different alloy steels made by the process, the alloys being nickel, chromium, vanadium, etc.

Bessemer Process. The Bessemer converter consists of a cylindrical cast-iron vessel lined with firebrick and having a conical-shaped mouth-piece. The bottom consists of a casting provided with a series of tuyeres or air ports through which a blast of air is admitted.

The pig iron is brought from the blast furnace in a molten state and placed in a large mixer, holding 200 to 300 tons, heated by oil or gas. The converter is placed in a horizontal position and a charge of molten metal admitted. The blast is turned on and the converter put in a vertical position. The air blowing up through the molten iron burns out the carbon and other impurities. This requires from ten to twenty minutes. During this period considerable of the iron is blown away. The quality of the product is determined by observation of the flame emerging from the converter. After all impurities have been burned out a small amount of "spiegeleisen" is admitted in order to bring up the carbon to the desired content. At the end of the "blow" the converter is placed in a horizontal position, the blast shut off, and the metal poured into a large ladle, from which it is poured into ingots. The production of steel by the Bessemer process is being replaced by the open-hearth method.

Open-hearth Process. Fig. 21 shows a typical form of furnace used in the manufacture of steel. This consists of an open hearth *K*, lined with either dolomite (limestone) or silica (sand), depending upon the per cent of phosphorus in the material constituting the charge. Producer gas is used as the fuel, and is preheated by passing through the "regenerative" chamber *F*. The air passes through the chamber *E*, where it is also preheated. The air and gas combine in chamber *H*, producing a very high temperature over the hearth. The depressed roof causes the flame to *impinge* on the charge. The products of combustion pass out through the chamber *I* to the chambers *E* and *F*, which are filled with layers of firebrick. These chambers become very hot. At

the end of a given interval of time, usually twenty to thirty minutes, a set of valves are operated which cause the air and gas to come in through the right-hand chambers and pass out through the left-hand chambers. In this way the waste heat in the products of combustion is utilized in preheating the air and gas.

The charge consists of all kinds of steel and iron scrap together with a certain amount of flux, depending upon the

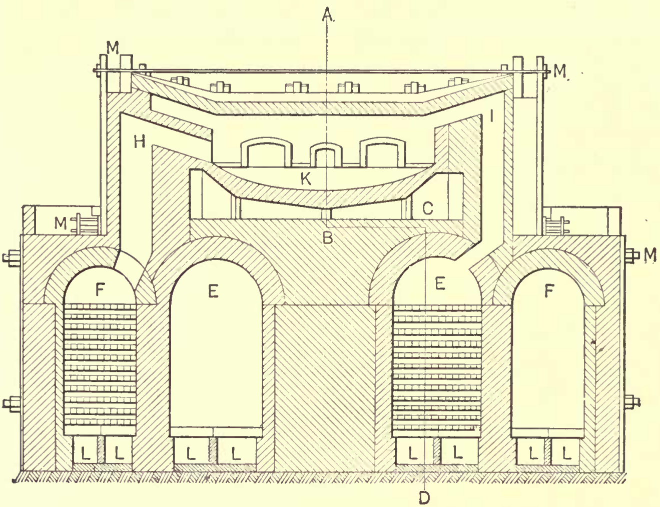


FIG. 21.

character of the scrap. In other cases the pig iron is taken directly from the blast furnace and run into the mouth of the open-hearth furnace. This process takes from five to six hours. The hearth holds from 50 to 100 tons of metal. The per cent of carbon can be definitely fixed in this process, as samples can be drawn off from time to time and tested. At the end of the heat the steel is drawn off in a large ladle from which it is poured into large ingots, which are later put

through the rolls and formed into the various structural shapes such as rails, I beams, angles, etc. The open-hearth process gives a steel of more uniform composition, and for this reason is rapidly replacing the Bessemer process.

The physical properties of steel depend upon the method of manufacture and upon the chemical composition of the steel. The carbon content has much to do with the hardness of the steel. Silicon increases the strength and reduces the ductility of steel. Manganese increases the hardness and raises the elastic limit and ultimate strength. Phosphorus increases the static strength, but reduces the resistance to shock. Table V shows the composition of steel plates used for various purposes:

TABLE V

Quality.	Carbon.	Manganese.	Sulphur.	Phosphorus.
Firebox.......	.16	.35 to .50	Not over .04	Not over .02
Boiler........	.18	.35 to .60	Not over .045	Not over .04
Flange.......	.18	.35 to .60	Not over .045	Not over .04
Ship.........	.15	.35 to .65	Not over .060	Not over .08
Tank........	.10	.40	Not over .100	Not over .12

PROB. 37. A steel bar $1\frac{1}{2}$ ins. in diameter is subjected to a tensile pull of 18,000. Find the unit tensile stress and the factor of safety.

ART. 15. TIMBER, STONE, AND BRICK

Timber varies more widely in general characteristics than any of the common building materials. The structure of wood affords the only positive means of identifying the different kinds. Color, weight, and smell are all functions of the structure of the wood. In the lumber trade timber is divided into two classes, "hard woods" and "soft woods." The terms "fine grained," "coarse grained," "straight

grained," and "cross grained," refer to the nature of the annual rings, whether they are narrow or wide, and to the fibers, whether they are parallel or twisted relative to the axis of the limb or tree.

Timber when cut contains a high percentage of moisture, which is removed by drying the lumber in kilns which are heated by steam. Hard woods such as oak, ash, maple, etc., are air-seasoned from three to six months to allow a gradual shrinkage before being finally placed in the kiln. Seasoning increases the strength of timber, but once seasoned, timber should not be exposed to the weather. Knots reduce the strength of timber. If possible the knotty side should be in compression. Table VI gives the strength of the more common woods. These values are average only and may vary, depending upon the character of the timber. Generally the denser, close-grained woods are the stronger. A study of the fracture of the various woods when subjected to compression will help the student to classify timber by its structure. The elastic limit is not well defined, especially in transverse tests. For a complete discussion of timber the student is referred to Johnson's "Materials of Construction."

TABLE VI

Material.	Weight, Lbs. per Cu. Ft.	Ultimate Strength.		Shear Across Grain.
		Tensile.	Compressive.	
Yellow Pine (Georgia)	38	12,000	7,000	5,000
White Pine	24	7,000	5,500	2,000
White Oak	50	12,000	7,000	4,000
Maple	43	18,000	7,500	
Cypress	28.7	6,000	5,000	
Ash	40	15,000	8,000	
Chestnut	41.2	8,500	4,000	2,000
Hemlock	25	6,000	7,000	2,500

Stone used for structural purposes should possess the qualities of cheapness, durability, strength, and beauty. As a general rule the densest, hardest, and most uniform stone will meet these requirements. The stone on fracture should be bright, clean, and sharp without loose grain and free from any dull, earthy appearance.

The crushing strength of stone is found by placing a prism or cube of the material in a testing machine. The specimen will generally fail by shear on a well-defined angle. This is typical of all brittle materials. Geological classification divides rocks into three groups—igneous, metamorphic, and sedimentary. Greenstone, trap, and lava are examples of igneous rocks; granite, marble, and slate, of metamorphic; sandstone and limestone of sedimentary.

Trap is the strongest of building stones, but is little used owing to its toughness and difficulty of quarrying. It is very durable.

Granite is the strongest and most durable of all the stones commonly used. It breaks with regularity and is easily quarried into simple shapes. Its extreme hardness and toughness makes it an expensive stone where special form is desired, hence it is most commonly used for foundations, piers, or for large buildings. The largest blocks of stone ever quarried have been of granite, one case being noted where a shaft 115 ft. long and 10 ft. square at the base and weighing 850 tons was quarried.

Limestone is composed chiefly of carbonate of lime. Limestones vary in color and composition, depending upon the character of the deposit.

Marble; any limestone which will take on a polish is called a marble. This makes a most beautiful building material and is used for interior decoration. Marble is extensively used in modern plumbing fixtures.

Sandstones are so called because they are made up chiefly of sand, cemented and consolidated. Sandstones

vary in color and texture, depending upon ingredients such as lime, iron, manganese, and alumina. The durability of the sandstones varies with their physical and chemical properties. Sandstones are as resistant to weather as granite and are less affected by fire. They are easily wrought into shape with hammer and chisel and, therefore, constitute the most common stone used for building purposes.

The weights and strengths of the various stones are given in Table VII:

TABLE VII

Material.	Weight, Lbs. per Cu. Ft.	Ultimate Strength.	
		Compression.	Shear.
Sandstone	150	5,000	1,400
Limestone	160	8,000	1,700
Granite	165	14,000	2,500
Trap	175	16,000	

The above values are the results of tests on small specimens. A test of an actual structure would show values of approximately 75 per cent of the above.

Brick is made of common clay, consisting mainly of silicate of alumina, by submitting it to a temperature which converts it into a semi-vitrified mass. Owing to its ease of manufacture, transportation, and handling, brick is used in place of stone. When properly made it is almost as strong as stone. Professor Baker gives the following six requisites for good brick:

1. A good brick should have plane faces; parallel sides, sharp edges and angles.
2. It should be of fine, compact, uniform texture; should be quite hard, and should give a clear ringing sound when struck a sharp blow.

3. It should not absorb more than one-tenth its weight of water.
4. Its specific gravity should be two or over.
5. The crushing strength of half brick when ground flat and pressed between thick metal plates should be at least 7000 lbs. per sq. in.
6. Its modulus of rupture should be at least 1000 lbs. per sq. in.

The common size of a brick in this country is $8\frac{1}{4}\times4\times2\frac{1}{4}$, thus making twenty-three to the cubic foot. These average about $4\frac{1}{2}$ lbs. each. Brick is sold and laid by the thousand.

When used as paving materials both brick and stone must be carefully tested for their abrasive properties.

PROB. 38. Find the weight of a stone pier whose lower base is 12×10; upper base 8×10 and altitude 6 ft.

PROB. 39. Find the weight of a 13-in. brick wall built in the shape of a triangle, whose base is 30 ft. and altitude 12 ft.

PROB. 40. Find the unit stress at the base of a granite shaft 100 ft. in height What is the factor of safety?

ART. 16. PROPERTIES OF MATERIALS

Wrought iron weighs 480 lbs. per cu. ft. This fact gives rise to the following rules for determining the weights of bars or other parts of uniform cross-section: A bar of wrought iron 1 yd. long and 1 sq. in. in cross-section weighs 10 lbs. Steel is 2 per cent heavier and weighs 10.2 lbs. per yard. Cast iron, being 6 per cent lighter, weighs 9.4 lbs. per yd. For example, a steel I beam having a cross-section of 5.88 sq. ins. weighs $5.88\times10.2=60$ lbs. per yd. A cast-iron grate bar 6 sq. ins. in cross-section weighs $6\times9.4=57.4$ lbs per yard. A wrought-iron boiler plate $\frac{3}{4}$ in. thick weighs $\frac{3}{4}\times1\times10$ ins. $=7.5$ lbs. per yard per inch width of plate. Table VIII gives the average weights in pounds per cubic foot of the various materials used in construction.

TABLE VIII

Material.	Weight, Lbs. per Cu. Ft.	Specific Gravity.
Steel	490	7.05
Wrought iron	480	7.69
Cast iron	450	7.21
Stone	160	2.60
Brick, pressed	140	
Brick, common	125	
Brass	500	8.1
Lead	710	11.38
Stone masonry	150	
Timber	40	.64

Prob. 41. A cast-iron water pipe is 24 ins. in diameter. The thickness of the metal is 1.5 in. Find the weight of the pipe per linear yard.

Prob. 42. The bed plate of planer weighs 450 lbs. per linear yard. Find its cross-sectional area.

Art 17. Review Problems

Prob. 43. Find the weight of a yellow pine beam 8×12 ins. cross-section and 20 ft. long.

Prob. 44. A steel wire 1.3 in. in diameter broke under a tensile load of 294,000 lbs. Find its ultimate tensile strength.

Prob. 45. A cast-iron bar 1 in. in diameter broke under a compressive load of 74,000 lbs. Find the ultimate compressive strength.

Prob. 46. A block of maple $2 \times 2 \times 2$ ins. broke under a compressive load of 24,000 lbs. Find the ultimate strength.

Prob. 47. A boiler plate specimen $\frac{1}{2} \times 2$ ins. broke under a load of 55,000 lbs. The elongation in 8 ins. was 2.5 ins. Find the ultimate strength and the per cent elongation.

Prob. 48. A stone pier is 8×10 ft. in cross-section. What safe load can the pier carry if the safe bearing power of the soil on which the pier rests is 2.5 tons per sq. ft.

Prob. 49. In a tension test on a steel specimen 0.8 in. in diameter the total elongation was found to be .00264 in. under a load of

5000 lbs. Find the modulus of elasticity, if the part under test was 8 ins. long.

PROB. 50. A hollow steel crank shaft is 10 ins. outside diameter and 7 ins. inside diameter. Find its weight if the shaft is 12 ft. long.

PROB. 50*a*. The diameter of a steel bar was 0.8 in. before testing and 0.56 in. after testing. Find the reduction in area.

PROB. 50*b*. Find the safe load that can be carried by a brick pier 24×60 ins., using a factor of safety of 12.

CHAPTER IV

THEORY OF BEAMS

ART. 18. BEAM REACTIONS

BEAMS are divided into two general classes—simple and cantilever, depending upon their method of support. A beam supported at both ends is called a *simple* beam, while a cantilever beam is supported at one end only. The force exerted by the part supporting the beam is called the reaction of the support, or more commonly the *beam reaction.* These reactions are determined by applying the principle of moments discussed in Art 2, Chapter I.

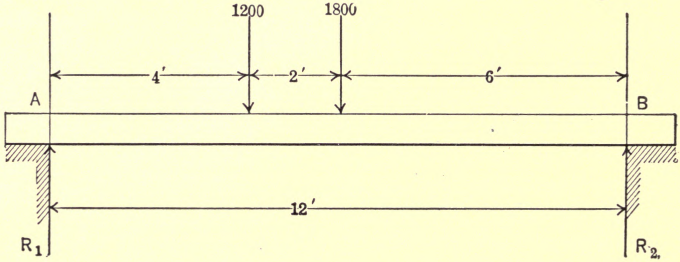


FIG. 22.

AB, Fig. 22, represents a simple beam supported at *A* and *B*, loaded as shown. Let R_1 equal the reaction of the left-hand support, and R_2 the reaction of the right-hand support. The value of R_1 is found by taking moments about the point R_2, thus:

$$-6\times1800+8\times1200+12R_1=0;$$
$$12R_1=20400;$$
$$R_1=1700.$$

In like manner R_2 is found by taking moments about R_1, thus:

$$4\times1200+6\times1800-12R_2=0;$$
$$12R_2=15600;$$
$$R_2=1300.$$

The accuracy of these results can be checked from the condition of equilibrium that the algebraic sum of the vertical forces equal zero, thus:

$$+R_1-1200-1800+R_2=0;$$
$$1700-1200-1800+1300=0;$$
$$0=0.$$

In all problems dealing with the reactions of simple beams proceed in the following manner: *First,* take moments about the left-hand support to determine the right-hand reaction; *second,* take moments about the right-hand support to determine the left-hand reaction; *third,* check results by equating the algebraic sum of the vertical forces on the beam equal to zero. In most cases the weight of the beam must be included in figuring the reaction. A beam weighing 60 lbs. per lin. ft. is loaded as shown in Fig. 23. To find the reactions in this

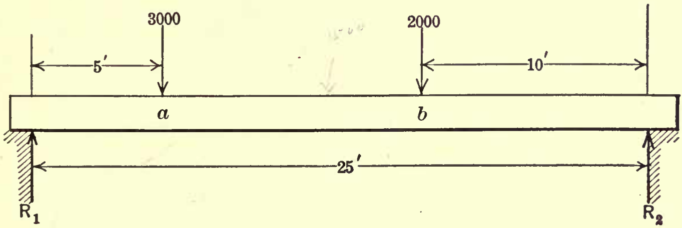


FIG. 23.

case assume the weight of the beam equal to a force of $25 \times 60 = 1500$ lbs. concentrated at the center of the beam. Taking moments about R_1 there results,

$$3000 \times 5 + 1500 \times 12\tfrac{1}{2} + 2000 \times 15 - 25R_2 = 0;$$
$$R_2 = 2550.$$

Taking moments about R_2 gives

$$-2000 \times 10 - 1500 \times 12\tfrac{1}{2} - 3000 \times 20 + 25R_1 = 0;$$
$$R_1 = 3950.$$

Using $\Sigma P = 0$ as a check, there results,

$$R_1 - 3000 - 1500 - 2000 + R_2 = 0,$$

or

$$3950 - 6500 + 2550 = 0;$$
$$6500 - 6500 = 0.$$

When a beam overhangs the one support the reactions can be found just the same as in the previous example. Consider a beam weighing 40 lbs. per lin. ft., loaded as shown in Fig. 24. Assume the total weight $= 40 \times 30 = 1200$ lbs. con-

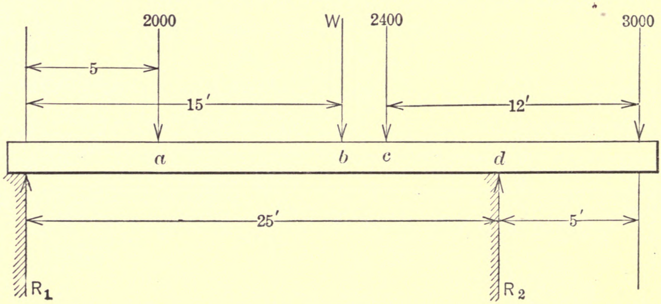


Fig. 24.

centrated at the center of the beam. To find the value of R_2 take moments about R_1 thus:

$$5 \times 2000 + 15 \times 1200 + 18 \times 2400 - 25R_2 + 30 \times 3000 = 0;$$
$$R_2 = 6448.$$

Likewise, taking moments about R_2 gives,

$$+5\times3000-7\times2400-10\times1200-20\times2000+25R_1=0;$$

$$R_1=2152$$

PROB. 51. A simple beam of 25-ft. span carries loads of 300, 250, and 400 lbs. distant 5-ft., 10-ft., and 20-ft. respectively from the left end. Beam weighs 30 lbs. per lin. ft. Find the reactions of the supports.

PROB. 52. A simple beam of 20-ft. span carries loads of 5000, 3000, and 4000 lbs. located 4-ft., 10-ft., and 12-ft. from the left end. If the beam weighs 35 lbs. per lin. ft., find the reaction of the supports.

PROB. 53. A beam 28 ft. long overhangs the right-hand support a distance of 7 ft. The beam carries a load of 6000 lbs. at the extreme right end and also loads of 2000 and 3000 lbs. located 8 ft., and 12 ft. from the left end. Find the reaction of the supports.

ART. 19. BENDING MOMENT

It is apparent that the tendency of the external forces, or loads, acting on a beam will be to cause a flexure or bending of the beam. This tendency increases with the external loads and the span of the beam.

The *bending moment* at any section of a beam is defined as the *algebraic sum* of the moments of *all* the forces acting to the left of the given section. The bending moment at the left-hand support equals zero, as there are no forces to the left of this section. The bending moment at the right-hand support equals zero in order to maintain the condition of equilibrium. Between the two reactions the bending moment will vary. The point in the beam at which the bending moment is the greatest or a " maximum " is called the *dangerous section* of the beam, for at this point the internal stress is also a maximum. The method of calculating this bending moment is best shown by a series of typical problems.

Example. Case I. Weight of beam neglected. Assume a beam loaded as shown in Fig. 25. By taking moments about the supports the reactions are found to be $R_1 = 2600$,

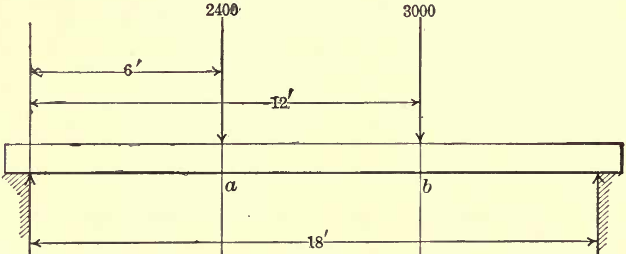


Fig. 25

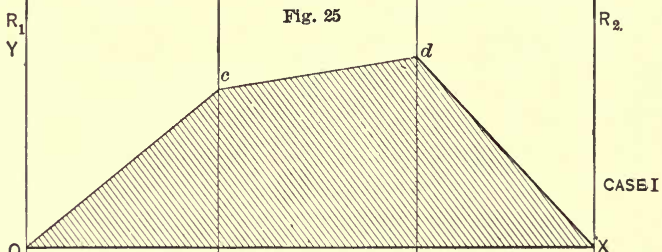


Fig. 26

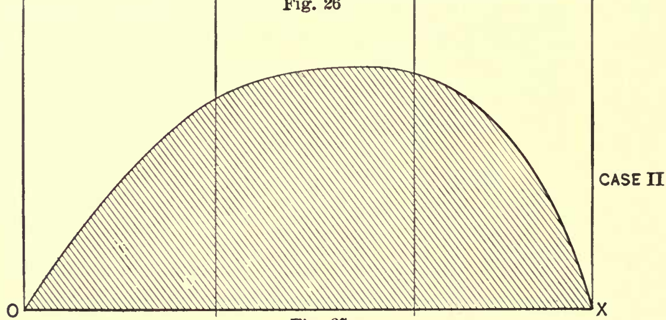


Fig. 27

$R_2 = 2800$. The bending moment at the points $a = 6 \times R_1 =$ 15,600 ft.-lbs. The bending moment at point $b = 12R_1 - 6 \times 2400 = 16{,}800$ ft.-lbs. In Fig. 26 let distances measured along the vertical ϕY equal the bending moments to scale.

The line *ocdx* shows the variation in the bending moment at the various points in the span of the beam. To clearly show this variation it is usual to cross-section the area included between the curve and the horizontal.

CASE II. Assume beam weighs 30 lbs. per lin. ft. Each reaction will be increased by an amount equal to one-half

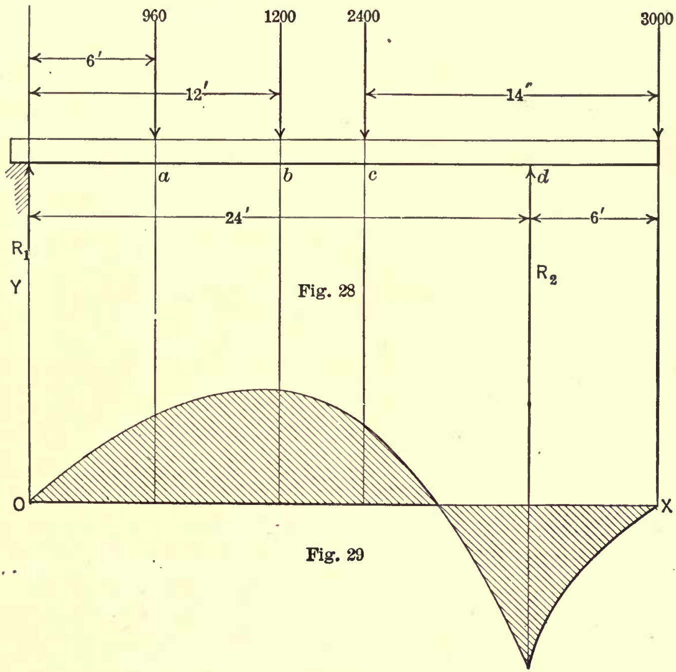


Fig. 28

Fig. 29

the total weight of the beam or 270 lbs., hence $R_1=2870$ and $R_2=3070$.

The weight of the section between the point a and the reaction R_1 can be considered as a single load equal to $30\times 6 = 180$ lbs. concentrated at the center of the section, assuming that the beam is of uniform cross-section. The forces acting to the left of the point a are the reaction R_1, and the weight

of the 6-ft. section of the beam concentrated at a distance of 3 ft. from the point a. The bending moment at point $a=6\times R_1-190\times 3=16{,}680$ ft.-lbs. The forces acting to the left of the point b are the reaction R_1, the load of 2400 lbs. and the weight of the 12-ft. section of the beam concentrated at a distance of 6 ft. from the point b. Hence the bending moment at point $b=12\times R_1-6\times 2400-6\times 360=17{,}880$ ft.-lbs. Fig. 27 shows the bending-moment diagram for this case. Here it will be noted the outline of the diagram is a curve rather than a broken line as in Case I, where the weight of the beam was neglected.

Example. A beam 30 ft. long overhangs the right-hand support a distance of 6 ft. If the beam is loaded as shown in Fig. 28 and weighs 40 lbs. per ft., find the bending moments at the points a, b, c, and d, and draw the bending-moment diagram.

Solution. Taking moments about the reactions gives

$$R_1=1820 \text{ and } R_2=6940.$$

Bending moment at $a=6\times R_1-6\times 40\times 3=10200$ ft.-lbs.

Bending moment at $b=12\times R_1-12\times 40\times 6-960\times 6=13200$ ft.-lbs.

Bending moment at $c=16\times R_1-16\times 40\times 8-960\times 10-1200\times 4=9600$ ft.-lbs.

Bending moment at $d=24\times R_1-24\times 40\times 12-960\times 18-1200\times 12-2400\times 8=-18{,}720.$

The bending moments are plotted to scale as shown in Fig. 29.

Prob. 54. In Fig. 23 compute the bending moments at the points a, b, and at the center of the span. Draw a bending-moment diagram.

PROB. 55. In Fig. 24 compute the bending moments at the points *a*, *b*, *c*, and *d*, and draw the bending-moment diagram.

PROB. 56. A cantilever beam 8 ft. long and weighing 35 lbs. per ft., carries a load of 4000 lbs. 6 ft. from the support. Find the bending moment at the support and at sections distant 2, 4, and 6 ft. from the support.

ART. 20. GENERAL EQUATION FOR BENDING MOMENT

All problems dealing with bending moments of simple beams can be solved by a general rule which will now be formulated. Assume a beam loaded as shown in Fig. 30

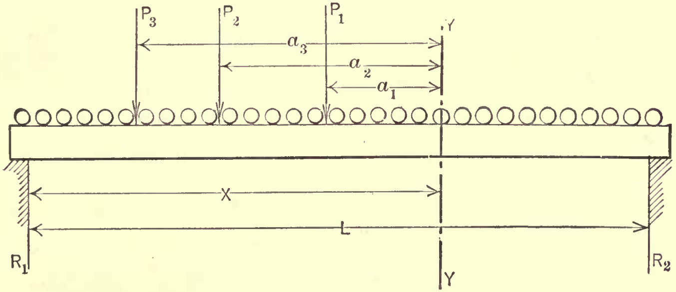


FIG. 30.

and let y–y be any section distant x feet from the left-hand reaction. Represent the distances from this section to the various concentrated loads P_1, P_2, P_3, by a_1, a_2, a_3, etc. Let w equal the uniformly distributed load in pounds per linear foot plus the weight of the beam in pounds per linear foot, provided the weight is known.

The uniform load on the x feet equals wx pounds, and this may be replaced by a single load of wx pounds located at the center of the x foot section or $\frac{x}{2}$ ft., from the section y–y. The bending moment at y–y equals the algebraic sum of the moments to the left of the section or is equal to the moment

of the left-hand reaction minus the moment of the concentrated loads, minus the moment of the uniform load to the left of y–y. The moment of the reaction $=+R_1X$. The moments of the concentrated loads$=P_1a_1-P_2a_2,-P_3a_3$, etc., briefly represented by $-P\Sigma a$ (read summation of all the terms of the form Pa.) The moment of the uniformly distributed load relative to the section

$$y-y=-wx\cdot\frac{x}{2}=-\frac{wx^2}{2}.$$

If M equals the bending moment at the section y–y it follows that

$$M=R_1x-\Sigma P\times a-\frac{wx^2}{2}. \quad . \quad . \quad . \quad . \quad (8)$$

The application of this general rule to determine the maximum bending moment of the four more common types of beams follow.

Case I. *Simple beam—concentrated load at the center, weight neglected*, see Fig. 31. Here $R_1=\frac{P}{2}$. By inspection

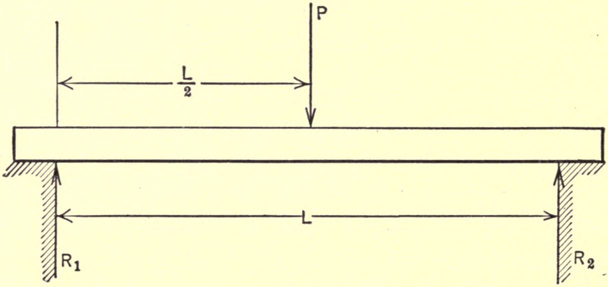


Fig. 31.

it is apparent that the bending moment will be a maximum at the center of the span, therefore $x=\frac{L}{2}$ and $W=o$, since the weight is neglected. Also $\Sigma Pa=O$, as there are no con-

centrated loads between the center of the beam and the left reaction. Substituting these values in Equation (8) there results,

$$M=\frac{P}{2}\times\frac{L}{2}-O-O=\frac{PL}{4}. \quad . \quad . \quad . \quad . \quad (9)$$

CASE II. *Simple beam—uniformly distributed load of* w pounds per lin. ft. see Fig. 32. In this case $R_1=\frac{wL}{2}$.

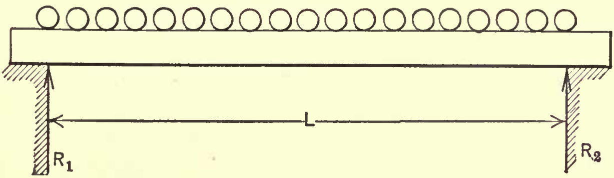


FIG. 32.

The bending moment will be a maximum at the center of the span, hence $x=\frac{L}{2}$. Substituting these values in Equation (8) gives

$$M=\frac{wL}{2}\cdot\frac{L}{2}-O-\frac{w}{2}\cdot\left(\frac{L}{2}\right)^2=\frac{wL^2}{8}. \quad . \quad . \quad . \quad (10)$$

Let W = the total uniformly distributed load $=wL$, then

$$M=\frac{(wL)\cdot L}{8}=\frac{WL}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

CASE III. *Cantilever beam—concentrated load at the end,* weight neglected, Fig. 33. In the case of cantilever beams the bending moment is negative and becomes a maximum at the point of support. Consider the beam supported at the right-hand side. Then $R_1=O$, $x=L$, and $w=o$. Substituting these values in the general Equation (8), there results

$$M=-O-PL-O=-PL \quad . \quad . \quad . \quad . \quad . \quad (12)$$

CASE IV. *Cantilever beam—uniformly distributed load* of w pounds per foot, Fig. 34. Here $R_1=O$; $x=L$; therefore

$$M=-O-O-\frac{wL^2}{2}=\frac{wL^2}{2}=\frac{WL}{2}. \quad . \quad . \quad . \quad (13)$$

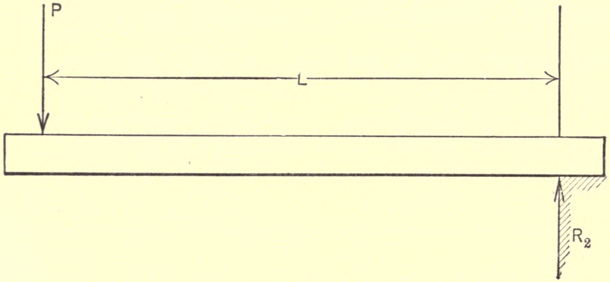


FIG. 33.

NOTE.—It is customary to represent concentrated loads by the letter P, and the total uniformly distributed load by W. The uniformly distributed load in pounds per linear foot is represented by w.

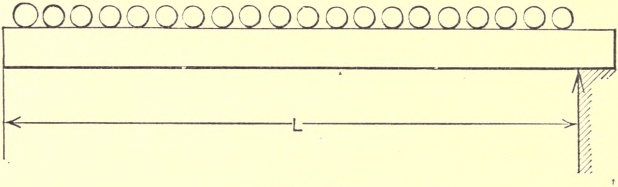


FIG. 34.

PROB. 57. Find the maximum bending of a simple beam where span is 18 ft. The beam carries a uniform load of 200 lbs. per lin. ft., and a concentrated load of 1800 lbs. 4 ft. from the left end.

PROB 58. Find the maximum bending moment on a cantilever beam which projects 10 ft. beyond its support and carries concentrated loads of 500 and 1500 lbs. located 4 and 8 ft. from the end of the beam.

Art. 21. Internal Resisting Moments

The internal bending moment produces a tension in the lower fibers of the beam and a compression in the upper fibers. As the beam bends the upper fibers shorten and the lower fibers lengthen. Somewhere between these two points there is a neutral plane which neither elongates nor shortens. It will be shown later that this neutral plane or axis passes through the gravity axis of the section of the beam.

It has been proven experimentally that the stress on any fiber of the beam varies directly with its distance from the neutral axis, provided the elastic limit of the material is not exceeded.

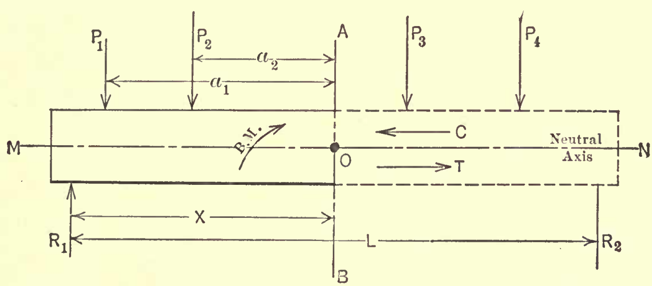


Fig. 35.

Assume a beam loaded as shown in Fig. 35. Let AB be the section where the bending moment is a maximum. Let MN be the neutral axis. All fibers above the axis MN will be in compression and all fibers below MN will be in tension. Now consider the right-hand section of the beam removed, and a horizontal force of C pounds equal to the total compressive stress in top part of beam, inserted; also a horizontal force of T pounds inserted to replace the total tensile stress in bottom fibers of beam. It is evident that the external bending moment tends to rotate the beam in a clockwise direction about the point O. To insure equilibrium the

forces C and T must produce a moment about O equal and opposite to the external bending moment. The moment exerted by the internal resistance of the beam is called the *internal resisting moment.*

Fig. 36 represents the cross-section of the beam at AB, and S_c equals the unit compressive stress on the outermost fiber of the beam in compression, and S_t equals the unit tensile stress on the outermost fiber of the beam in tension. The stress along the neutral line MN is zero. Take any plane CD at a distance of x units from the neutral axis. Let the thickness of this plane be very small and its cross-sectional area in square inches be represented by a. Represent the unit stress on this area by S_x and let the stress on the outermost fiber be represented by S. Then

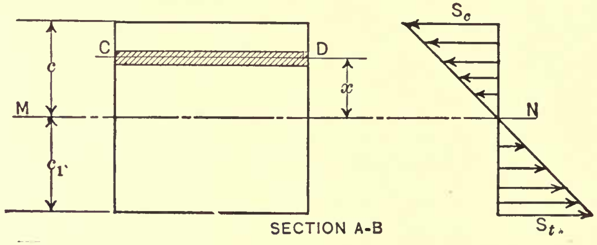


Fig. 36.

$$\frac{S_x}{S}=\frac{x}{c} \quad \text{or} \quad S_x=S\times\frac{x}{c}.$$

The total stress on the area a will equal the unit stress $\left(S\times\frac{x}{c}\right)$ times the area a, or equal $\left(\frac{S}{c}\times ax\right)$. This total stress on the area a exerts a resisting moment about the point O which tends to counteract the external bending moment. The resisting moment of the resistance of the area a equals

the total stress $\frac{S}{c}ax$ times the moment arm x, or equals

$$\frac{S}{c}ax \cdot x = \frac{S}{c} \times ax^2.$$

Now, consider the total cross-section of the beam at AB to be made up of an infinite number of small areas of the form a. Let N equal the number of these areas each equal to a in value and let A equal the total cross-section of the beam. Then it is evident that $A = aN$. The total resisting moment of the beam will equal the sum of the resisting moments of each elementary area, or the resisting moment equals

$$\frac{S}{c}ax_1^2 + \frac{S}{c}ax_2^2 + \frac{S}{c}ax_3^2 \quad . \quad . \quad + \frac{S}{c}ax_n^2$$

$$= \frac{S}{c}a[x_1^2 + x_2^2 + x_3^2 \quad . \quad . \quad + x_n^2]. \quad . \quad (14)$$

Let

$$(x_1^2 + x_2^2 + \quad . \quad . \quad . \quad + x_n^2) = NX^2,$$

where X^2 = the average of all the terms in this sum. Substituting this value in Equation (14) there results, the resisting moment equals

$$\frac{Sa}{c}[NX^2],$$

then since there are N terms it follows that

$$(x_1^2 + x_2^2 \quad . \quad . \quad . \quad + x_n^2) = NX^2;$$

but $aN = A$ (the total cross-section of the beam); hence the resisting moment $= \frac{S}{c}(AX^2)$. It is evident that the term AX^2 is entirely dependent on the form of cross-section of the beam. This factor might be called "the factor of strength," but is commonly referred to as the *moment of inertia* of

the section and is generally represented by I; or the resisting moment equals

$$\frac{SI}{c}. \qquad (15)$$

For equilibrium the internal resisting moment $\frac{SI}{c}$ must equal the external bending moment M, or there results the fundamental equation for the design of beams

$$M=\frac{SI}{c}. \qquad (16)$$

This equation can be written,

$$\frac{M}{S}=\frac{I}{c}. \qquad (17)$$

The term $\left(\frac{I}{c}\right)$ is called the "Section Modulus" of the beam. It is to be noted that the section modulus is entirely independent of the material of which the beam is made and depends only upon the distribution of the metal relative to the neutral axis of the section.

Prob. 59. Consider a rectangular beam of breadth b ins. and depth d ins. Divide the cross-section into 20 rectangles of breadth b ins. and depth $\frac{d}{20}$ in. Derive an expression for the resisting moment in terms of b, d, and the maximum fiber stress S.

Prob. 60. Find the total compressive stress and the total tensile stress on a rectangular beam whose depth is 12 ins. and breadth 8 ins. Stress on outer fiber equals 1000 lbs. At what point in the section is the stress equal to zero?

Art. 22. Center of Gravity

A body may be considered as made up of a series of small particles. The weights of all these particles form a system of vertical parallel forces, and the resultant of this system must

evidently equal the sum of these forces or the total weight of the body. The point at which this resultant force or weight acts is called the center of gravity of the body. As commonly defined the center of gravity of a body is the point through which the line of action of the weight of the body always passes.

In the following discussion the term center of gravity will be abbreviated to the symbol c.g. If a bar of uniform cross-section be suspended by a rope attached at one end of the bar, it is evident that the bar will hang in a vertical, and not in a horizontal, position; or, if the rope is attached to the center, the bar will probably assume a horizontal position. Hence a vertical line drawn through the point of suspension of a body must always pass through the c.g. of the body.

The c.g. of bodies which are symmetrical with respect to a given point, and are of uniform density, will be at the given point. Thus the c.g. of a sphere is at its geometrical center and the c.g. of a circle is at its center. The student can readily think of many more examples. Therefore, in many cases the determination of the c.g. is simply a matter of inspection. In the case of unsymmetrical figures the c.g. may be found experimentally by making a template of card-board to represent the body to a given scale. Next suspend the template, first from one point and then from another, and in each case draw a vertical line through the point of suspension. The intersection of these two vertical lines will locate the c.g. of the template, from which the c.g. of the body can readily be found. Take, for example, the counter-weight attached to the crank of certain forms of steam engines. Let the weight be of uniform thickness and its cross-section be of the form shown in Fig. 37, which is a template of the given weight. If the template be supported from the point O and the vertical line CD be drawn through the point of suspension, then the c.g. of the body will lie somewhere on this line. Likewise, if the template be suspended from the

point B the c.g. will lie on the line AB. Therefore, the c.g. of the body will be found at the point E of intersection of the two lines CD and AB. This method is frequently used in the drafting room, as no computations are necessary.

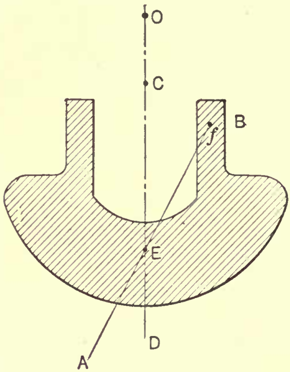


Fig. 37.

If a body can be divided into regular geometrical figures, such as triangles, squares, and rectangles, the c.g. is best obtained by applying the law that the moment of the resultant (which is the total weight of the body) is equal to the sum of the moments of the other forces (the weights of each of the separate figures).

In problems where the body is symmetrical with neither the vertical nor the horizontal axes the following general method must be applied to find the c.g. Assume a given body to be divided into a large number of very small parts, and the

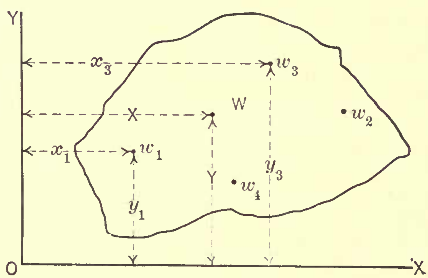


Fig. 38.

weights of each part to be represented by the points w_1, w_2, w_3, etc., as shown in Fig. 38. Locate each of these weights from the axes OX and OY, thus w_1 is x_1 units from the OY axis, and y_1 units from the OX axis. The resultant of all these

small weights will be the total weight W of the given body. Thus $W=w_1, w_2, w_3 \ldots$, etc. This weight or resultant will act at the c.g. of the body. Let this point be X units from the OY axis and Y units from the OX axis. Then the distance Y may be found by taking moments about the axis OX.

Thus,

$$w_1y_1+w_2y_2+w_3y_3+ \ldots =(w_1+w_2+w_3+)Y=W\times Y,$$

or

$$Y=\frac{w_1y_1+w_2y_2+w_3y_3+}{w_1+w_2+w_3+}. \quad \ldots \quad (18)$$

In like manner the value of X may be found by taking moments about the OY axis. Thus,

$$X=\frac{w_1x_1+w_2x_2+w_3x_3+}{w_1+w_2+w_3+}. \quad \ldots \quad (19)$$

Fig. 39 shows the cross-section of a riveter frame. Locate the c.g. of this section. In dealing with problems of this kind the moments of the areas are taken to determine the location of the c.g., hence Equations (19) and (18) can be written

$$X=\frac{a_1x_1+a_2x_2+a_3x_3+}{a_1+a_2+a_3+}, \quad \ldots \quad (20)$$

$$Y=\frac{a_1y_1+a_2y_2+a_3y_3+}{a_1+a_2+a_3+}. \quad \ldots \quad (21)$$

This figure is symmetrical about the vertical axis and, hence, its c.g. lies somewhere on the line AB say at the point H, x inches from the line CD. Divide the section into the three rectangles I, II, and III. By inspection it is clear that the c.g. of each rectangle is at the point of intersection of the diagonals, and also that the weight of each part is proportional to the area of the cross-section. The moment of the weight of each section about any given point will then be proportional to the moment of the area.

The c.g. of part III is located $9\frac{1}{2}$ ins. from the line CD, of part II, $5\frac{1}{4}$ ins. from CD, and part I, $\frac{3}{4}$ in. from CD. The area of part I equals $5\frac{1}{2}\times 1\frac{1}{2}=8\frac{1}{4}$ sq. in., the area of part II equals $7\frac{1}{2}\times\frac{1}{2}=3\frac{3}{4}$ sq. in., and the area of part III equals

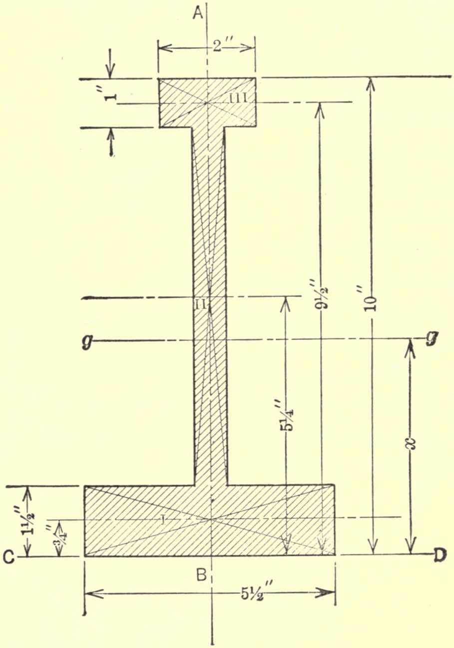


Fig. 39.

$2\times 1=2$ sq. in. Now, take moments of the areas I, II and III, about the line CD. The sum of these moments must equal the moment of the resultant (total area). Thus,

$$\tfrac{3}{4}\times 8\tfrac{1}{4}+3\tfrac{3}{4}\times 5\tfrac{1}{4}+9\tfrac{1}{2}\times 2=14\times x;$$

or

$$14x=44.87,$$

$$x=3.2 \text{ in.}$$

While the above method is accurate, errors are hard to locate if the equations contain many terms. In order to readily check results it is desirable to tabulate the data and results as shown in Table IX.

The c.g. of the cross-section of any compound shape can quickly be located by this scheme. Divide the section into rectangles and triangles; locate the c.g. of each section with respect to the base of the figure. Mark each section, give its dimensions, area, moment arm, and moment. The sum of the moment column (5) in Table IX divided by the total area as given in column (3) will give the distance X from the base of the figure to its c.g.

TABLE IX

Computations for Center of Gravity

1 Mark.	2 Dimensions, Inches.	3 Area, Sq. In.	4 Moment Arm, In.	5 Moment.
I	$5\frac{1}{2} \times 1\frac{1}{2}$	$8\frac{1}{4}$	$\frac{3}{4}$	6.18
II	$7\frac{1}{2} \times \frac{1}{2}$	$3\frac{3}{4}$	$5\frac{1}{4}$	19.70
III	2×1	2	$9\frac{1}{2}$	19.00
		14		44.88
			$x=$	3.2

The elementary areas or weights are in equilibrium with respect to the gravity axis. In Equation (20) $X=O$, with reference to the gravity axis, and it follows that $a_1x_2+a_2x_2+a_3x_3+\ .\ .\ .\ =0$ or $\Sigma ax=0$. That is, the summation of each elementary area times its distance from the gravity axis equals 0.

In Fig. 36 the total stress on any element of area equals $a \cdot x \cdot \dfrac{S}{c}$. The line MN represents the neutral axis. From the conditions of equilibrium it is evident that the sum of

the tensile stresses below the neutral axis must equal the sum of the compressive stresses above the neutral axis, and that the algebraic sum of all the stresses relative to the neutral axis equals 0. The summation of the stresses on all the elements of area equals $\frac{S}{c}\Sigma ax=0$, hence $\Sigma ax=0$, therefore the neutral axis of any beam is coincident with the gravity axis of the section.

PROB. 61. Locate the gravity axis of the vertical section of the 150-ton punch frame shown in Fig. 46.

PROB. 62. Locate the gravity axis parallel to the longer leg of a 5×4-in. angle; thickness of each leg equals $\frac{1}{2}$ in.

PROB. 63. Locate the gravity axis of a 12-in. channel section; average thickness of flange equals 0.5 in.; thickness of the web equals .28. Depth of flange 2.94 in.

PROB. 64. Locate the gravity axis of a T bar, whose depth is 4 ins., width of flange 4 ins., average thickness of flange $\frac{3}{8}$ in.; average thickness of stem equals $\frac{3}{8}$ in.

ART. 23. MOMENT OF INERTIA

In the formula for the internal resisting moment of a beam the factor $I=\Sigma ax^2$, is called the moment of inertia of the section, but as stated before, this term should be considered only as a " factor of strength." Its numerical value depends on the distribution of the metal in the beam relative to the neutral axis.

The *moment of inertia* is the quantity obtained by multiplying each elementary area of a given section by the square of its distance from the axis about which the moment is desired. Moment of inertia of various sections can be added and subtracted provided they are figured about a common axis. To determine accurately an expression for the moment of inertia of the common sections such as rect-

angles, squares, triangles, etc., involves the use of the calculus, so in this text no such values will be derived, but simply stated; thus the moment of inertia of a rectangle about its gravity axis equals $\frac{bd^3}{12}$, where b equals the breadth in inches, and d equals the depth in inches. The moment of inertia of a triangle about its gravity axis equals $\frac{bh^3}{36}$, where b equals the base and h equals the altitude. For a circle $Ig=\frac{\pi d^4}{64}$. For a square $Ig=\frac{d^4}{12}$.

In many cases it is desirable to transfer the moment of inertia from the gravity axis to some other axis which is

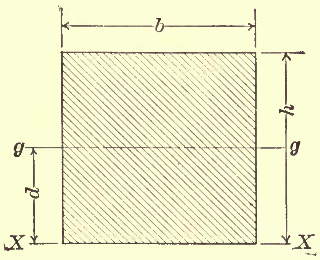


FIG. 40.

parallel to the given axis. The following formula shows the relation between the moment of inertia about the gravity axis g–g (see Fig. 40), and any axis (x–x) parallel to the gravity axis.

$$I_x=Ig+Ad^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where

I_x = Moment of inertia about given axis;

I_g = Moment of inertia about gravity axis;

A = the area of the section in square inches;

d = distance in inches between the parallel axes.

One of the more common applications of this reduction formula is in the case of the rectangle. Thus in Fig. 40 the value of $Ig=\frac{bh^3}{12}$; $A=bh$; $d=\frac{h}{2}$; Ix = moment of inertia about the base, hence

$$I_x=Ig+Ad^2=\frac{bh^3}{12}+bh\cdot\frac{h^2}{4}=\frac{bh^3}{3}.$$

For a hollow rectangle $Ig=\frac{bh^3}{12}-\frac{b_1h_1{}^3}{12}$ (See Fig. 41.)

For the I section $Ig=\frac{bh^3}{3}-\frac{(b-t)(h-2t_1)^3}{12}$. (Fig. 42.)

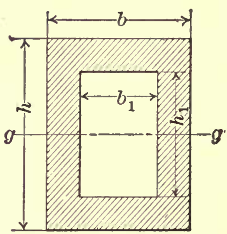


FIG. 41.

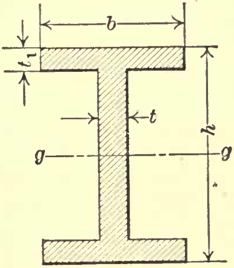


FIG. 42.

In any section which is not symmetrical with respect to the gravity axis, the moment of inertia can readily be found by dividing the section into rectangles located above and below the gravity axis. In each case the base of these elementary rectangles must be coincident with the gravity axis of the figure. For the T section the values of c and c_1 must be determined as explained in Art. 22. In Fig. 43 the moment of inertia of the T section is found by taking the moment of inertia of the part below the gravity axis, which equals $\frac{t_1c_1{}^3}{3}$, and adding the moment of inertia of the part

above the gravity axis, which equals $\frac{bc^3}{3} - \frac{(b-t_1)\ (c-t)^3}{3}$.

The value $\frac{t_1c_1{}^3}{3}$ is used because the base of the rectangle is coincident with the gravity axis of the entire figure. Hence the moment of inertia of a T section equals

$$\frac{t_1c_1{}^3}{3} + \frac{bc^3}{3} - \frac{(b-t\ )(c-t)^3}{3}.$$

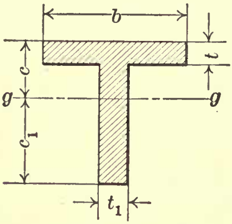


FIG. 43.

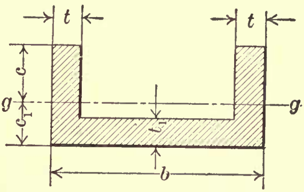


FIG. 44.

For a channel section (see Fig. 44) whose breadth is b, depth h, thickness of flange t, thickness of web t_1, the value of

$$Ig = \frac{2tc^3}{3} + \frac{bc_1{}^3}{3} - \frac{(b-2t)(c_1-t)^3}{3}$$

For the angle (Fig. 45) whose depth is h, thickness leg t, the value of

$$Ig = \frac{tc^3}{3} + \frac{hc_1{}^3}{3} - \frac{(h-t)\ (c_1-t)^3}{3}.$$

These values are correct in the case of cast-iron section. Standard steel shapes do not have parallel edges, hence the above formulæ do not hold. For exact values the student is referred to handbooks on structural steel.

For compound sections the value of Ig is determined by dividing the section into a series of rectangles and finding the value of the moment of inertia of each rectangle with

respect to the gravity axis of the figure. It is advisable to use a standard form for tabulating results as shown in Table X; assuming that the gravity axis has been located as explained in Art. 22.

To use this method the figure whose moment of inertia is to be found, is divided into a series of rectangles, each of which has its base coincident with the gravity axis of the figure. These rectangles are marked I, II, III, etc. These marks are placed in column 1, Table X. Column 2 gives the breadth of the rectangle. Rectangle I is considered as the difference of two rectangles, each having a breadth of

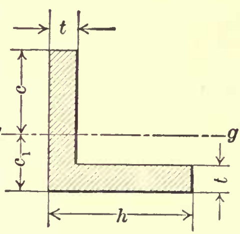


Fig. 45.

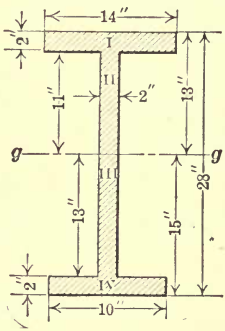


Table X.

14 ins. and depths of 13 ins. and 11 ins. respectively. Column 3 gives the greater depth, designated h_1; and column 4 gives the lesser depth designated h_2. Column 5 gives the values of h_1^3. These values are quickly determined by the use of a table of cubes of numbers. Column 7 gives the values of $(h_1^2 - h_2^3)$. Column 8 gives the values of $b \times (h_1^3 - h_2^3)$. The values in column 8 will be three times the moment of inertia of the given rectangle, as the moment of inertia of a rectangle about its base is bh^3; hence the final total of column 8 is divided by 3 to give the moment of inertia of the

entire figure. This method eliminates the necessity of using the transformation formula.

TABLE X

1 Mark	2 b	3 h_1	4 h_2	5 h_1^3	6 h_2^3	7 $(h_1^3-h_2^3)$	8 $b\times(h_1^3-h_2^3)$
I	14	13	11	2,197	1,331	866	12,124
II	2	11	0	1,331	0	1,331	2,662
III	2	13	0	2,197	0	2,197	4,394
IV	10	15	13	3,375	2,197	1,178	11,780
						3	30,960
						$I_g=$	10,320

Prob. 65. Find the moment of inertia about the gravity axis for the section shown in Fig 46.

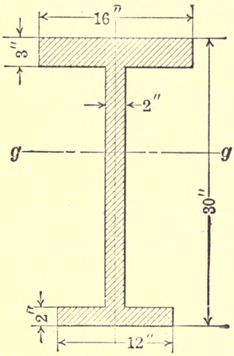


Fig. 46.

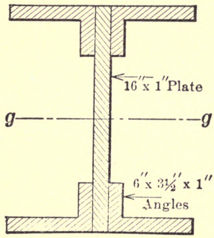


Fig. 47.

Prob. 66. A plate girder is constructed as shown in Fig. 47. Find the moment of inertia of the section with respect to the gravity axis.

Prob. 67. (*a*) Find the moment of inertia about the gravity axis of a circle 3 ins. in diameter. (*b*) Find the moment of inertia of the circle about an axis parallel to and located 8 ins. from the gravity axis.

Art. 24. Review Problems

Prob. 68. Locate the gravity axis of a standard 12-in. channel weighing 20.5 lbs. per foot. For dimensions of section see Cambria handbook.

Prob. 69. A beam 30 ft. in length overhangs each support. The left-hand support is located 4 ft. to the right of the extreme left end of the beam. Distance between supports is 20 ft. Beam carries concentrated loads of 2000, 3000, 4000, and 2000 lbs. located 0, 5, 10, 15, and 30 ft. from the left end respectively. Draw the bending-moment diagram.

Prob. 70. Fig. 48 shows a cast-iron bracket. Find the moment of inertia of the section AB with respect to the gravity axis.

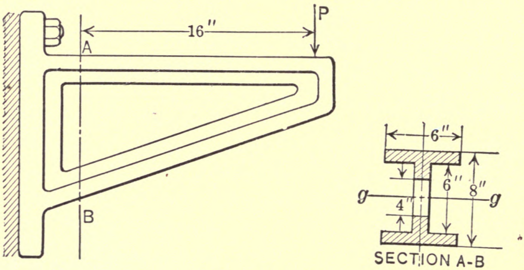


Fig. 48.

Prob. 71. Find the moment of inertia relative to the gravity axis of the box girder shown in Fig. 49, if the plates are 14 ins. in width.

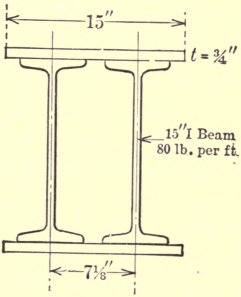


Fig. 49.

PROB. 72. Fig. 50 shows the cross-section of a crane hook. Locate the gravity axis of the section and find the moment of inertia of the section relative to the gravity axis.

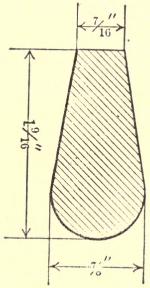


FIG. 50.

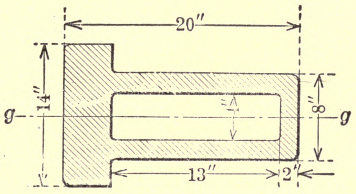


FIG. 51.

PROB. 73. Fig. 51 shows the cross-section of a riveter frame. Find the moment of inertia of the section relative to the gravity axis, g–g.

PROB. 74. A simple beam is loaded as shown in Fig. 52. Draw the bending-moment diagram.

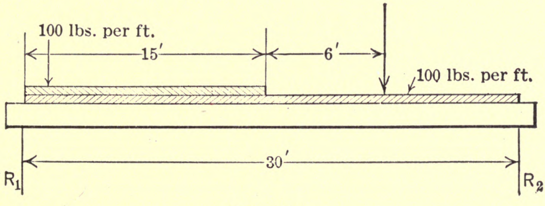


FIG. 52.

PROB. 75. Find the moment of inertia of an 8×12-in. wooden beam, about its horizontal and vertical gravity axes.

CHAPTER V

DESIGN OF BEAMS

ART. 25. SAFE LOADS—WOODEN BEAMS

IN designing beams, especially when used for the support of floors, two kinds of loads are recognized. *Live* loads consist of the weight of carriages, cranes, or other handling devices and their supported loads, machinery, merchandise, persons, or other moving objects. *Dead* loads consist of the actual weight of the structure itself with the walls, floors, partitions, roofs, and all other permanent fixtures.

Wooden beams are used for the support of floors; they are generally rectangular in shape and supported at the ends by either steel or wood girders. The spacing of the floor beams depends upon the span and the load carried in pounds per square foot by the floor. This load is fixed by the character of the building. The safe unit working stress for wooden beams is relatively low, owing to the necessity of using a high factor of safety on account of the non-uniform structure of the wood.

The formula $S=\frac{Mc}{I}$ is the basis for determining the safe load on any beam. For wooden beams of rectangular cross-section this formula may be stated in terms of the external load, the breadth and depth of the beam, and the span. Consider the case of a simple wooden beam carrying a concentrated load of P pounds at the center of a span of L inches. Let b equal the breadth and d equal the depth of the cross-

section of the beam. From Equation (9) the maximum bending moment is $\frac{PL}{4}$; the moment of inertia $I=\frac{bd^3}{12}$, and $c=\frac{d}{2}$ (gravity axis being coincident with neutral axis at the center of the section). Substituting these values in equation (16) gives

$$\frac{PL}{4}=S\cdot\frac{\frac{bd^3}{12}}{\frac{d}{2}}.$$

or

$$P=\frac{2Sbd^2}{3L}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$S=\frac{3}{2}\cdot\frac{PL}{bd^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

In Equations (23) and (24) the values of b, d, and L are to be expressed in inches. Equation (23) shows that the load varies as the breadth of the beam and as the square of the depth, which is the reason for usually having the depth of a beam greater than its breadth. For example, assume the case of a yellow pine floor beam 3×8 ins. in cross-section and 12-ft. span. Let $S=1000$ lbs. per sq. in.; then $b=3$ in., $d=8$ in., $S=1000$, and $L=12$ ft. $=144$ ins., hence

$$P=\frac{2}{3}\times\frac{1000\times3\times64}{144}=890 \text{ lbs.}$$

Upon reference to structural hand books it will be noted that there is a fixed maximum and minimum span for wooden beams of any given depth.

If the load is uniformly distributed the maximum bending

moment equals $\frac{WL}{8}$. Substituting this value in Equation (16) gives

$$\frac{WL}{8}=\frac{\frac{Sbd^3}{12}}{\frac{d}{2}} \quad \text{or} \quad W=\frac{4Sbd^2}{3L}, \qquad . \ . \quad (25)$$

and

$$S=\frac{3}{4}\frac{WL}{bd^2}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (26)$$

In the previous example, if the load is uniformly distributed instead of concentrated at the center of the span, the load is

$$\frac{4}{3}\frac{Sbd^2}{L}=\frac{4\times1000\times3\times64}{3\times144}=1780 \text{ lbs.}$$

The floors of cotton mills can generally be light, for the total weight of machinery, men, and materials will seldom exceed 30 lbs. per sq. ft. Rooms for pattern storage rarely carry more than 150 lbs. per sq. ft. Buildings for light machinery frequently have provision for loads of 250 to 300 lbs. per sq. ft.

Table XI gives the safe working stresses for various kinds of timber, based upon data recommended by the American Railway Association.

Prob. 76. A wooden beam 8×12 ins. in cross-section and having a span of 16 ft. carries a uniformly distributed load of 8000 lbs. Find the unit fiber stress and the factor of safety.

Prob. 77. A floor is supported by wooden beams 3×8 ins. in cross-section, having a span of 12 ft. If the beams are spaced 18 ins. center to center, find the safe load that can be placed on the floor in pounds per square foot.

Prob. 78. A wooden beam having a span of 16 ft., carries a concentrated load of 2160 lbs. at a point 10 ft. from the left end of the beam. Assuming that the depth of the beam equals $1\frac{1}{2}$ the breadth, find the dimensions of beam, using a safe working stress of 1200 lbs. per sq. in.

TABLE XI

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES, IN POUNDS PER SQUARE INCH

Kind of Timber.	Tension.		Compression.			Transverse.		Shearing.	
	With Grain.	Across Grain.	With Grain. End Bearing	With Grain. Column Under 15 Diams	Across Grain.	Extreme Fiber Stress.	Modulus of Elasticity.	With Grain.	Across Grain.
Factor of Safety.	Ten.	Ten.	Five.	Five.	Four.	Six.	Two.	Four.	Four
White oak	1,200	200	1,400	1,000	500	1,200	750,000	200	1,000
White pine	700	50	1,100	700	200	700	500,000	100	500
Southern long-leaf or Georgia yellow pine	1,200	60	1,400	1,000	350	1,200	750,000	150	1,250
Douglas fir	800	...	1,100	900	200	800	750,000	130	....
Short-leaf yellow pine	900	50	1,200	900	250	1,000	600,000	100	1,000
Red pine (Norway pine)	800	50	1,000	800	200	800	565,000	...	....
Spruce and eastern fir	800	50	1,200	800	200	700	600,000	100	750
Hemlock	600	...	....	800	150	600	450,000	100	600
Cypress	600	...	1,000	800	200	800	450,000	...	....
Cedar	700	...	1,100	700	200	700	350,000	100	400
Chestnut	850	...	....	800	250	800	500,000	150	500
California redwood	700	...	....	800	150	750	350,000	100	....
California spruce	....	...	....	800	....	800	600,000	...	....

Art. 26. Shear Diagrams

The vertical shear at any point in a beam is equal to the algebraic sum of all the forces acting to the left of the section. The vertical shear will be the maximum at one of the reactions of the beam. Therefore the internal stress at the reaction is one of pure shear. The cross-section of the beam at this point must be such that the unit shearing stress shall be great enough to insure the proper factor of safety. Let V equal the maximum vertical shear in pounds; A equal the cross-section of the beam in square inches and S equal the safe working unit shearing stress, then:

$$V=AS, \qquad \ldots\ldots \qquad (27)$$

or

$$S=\frac{V}{A}. \qquad \ldots\ldots \qquad (28)$$

It is customary to figure the vertical shear at the reactions, under each concentrated load and at the center of the beam. These values are then plotted to scale forming a "shear diagram." The vertical shear at any point in a beam distant x feet from the left reaction is given by the equation.

$$V=R_1-\Sigma P-wx \qquad \ldots\ldots \qquad (29)$$

where R_1 equals the left-end reaction in pounds; ΣP equals the algebraic sum of the concentrated loads to the left of the section, and wx equals the uniformly distributed load including the weight of the beam.

Example. Given a beam loaded as shown in Fig. 53. Plot the shear diagram.

Solution. Taking moments about R_1 gives

$$5\times1000+10\times2000+15\times2400-25R_2=0,$$

or

$$R_2=2440.$$

In like manner taking moments about R_2 gives $R_1=2960$.

The vertical shear at $a = R_1 = +2960$. Consider a point a very short distance to the left of the point b. The shear at this equals R, since there are no vertical loads between points a and b (the weight of the beam is neglected in this case). Now consider a section a short distance to the right of the point b; the shear at this point will be $(R_1 - 1000)$ or 1960 lbs. It is customary to assume that the value of the

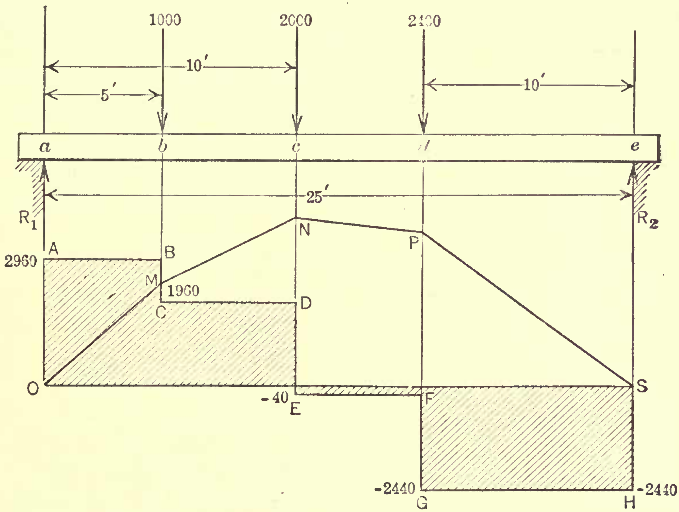


FIG. 53.

vertical shear changes at each concentrated load. Hence the shear at point

$$c = R_1 - 1000 - 2000 = 2960 - 3000 = -40 \text{ lbs.}$$

It will be noted that at the point c the vertical shear changes from a positive to a negative value; the shear at point $d = R_1 - 1000 - 2000 - 2400 = -2400$; the shear at $R_2 = -2440$ lbs. The curve $OABCDEFGH$, Fig. 53, shows at a glance the variation in the vertical shear.

In cases where both uniform and concentrated loads are involved the curve will be of a different form. In Fig. 54 the beam carries a uniformly distributed load of 200 lbs. per lin. ft., in addition to the concentrated loads indicated. By taking moments about R_1 and R_2 the reactions are found to be $R_1=4600$ and $R_2=4700$ lbs. To find the ver-

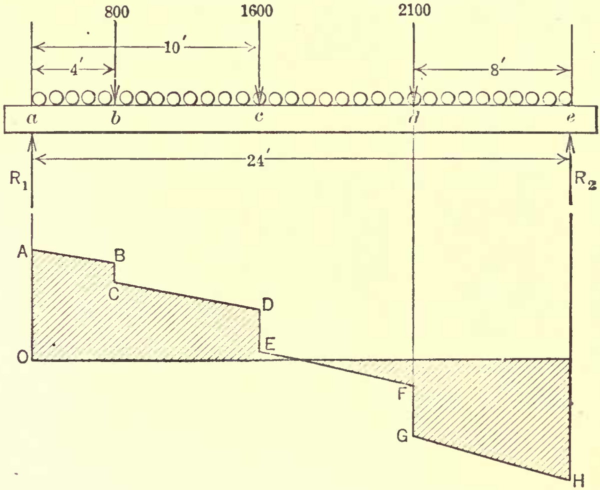


Fig. 54.

tical shear at the various points substitute in Equation (28) the various values, thus at point

a, $V=R_1-0-0=4600$ (here $x=0$);

b, $V=R_1-800-200\times x=4600-1600=3000$ (here $x=4$);

c, $V=R_1-800-1600-200\times 10=4600-4400=200$ (here $x=10$);

d, $V=R_1-800-1600-2100-200\times 16=4600-7700=-3100$ (here $x=16$).

The curve $OABCDEFGH$, Fig. 54, shows the variation of the vertical shear in this case.

The shear diagram is helpful in locating the dangerous section of a beam, for it can be proven that the *bending moment is always a maximum* at the point where the vertical shear passes through its zero *value.* In Fig. 53 it will be noted that the shear passes from a positive to a negative value at the point *c*, and, therefore, the bending moment will be a maximum at this point. The curve *OMNPS*, Fig. 53, shows the bending-moment diagram.

As a general rule beams will fail due to the internal stress caused by bending, hence if a beam is strong enough to withstand the bending action the vertical shear will take care of itself. However, it is well to investigate the unit vertical shear at the reactions. For example, in Fig. 53 the beam is $6 \times 9 = 54$ sq. ins. in cross-section. The vertical shear at $R_1 = 2960$, therefore the unit shear $= \frac{V}{A} = \frac{2960}{54} = 55$ lbs., and the factor of safety in shear $= \frac{3000}{55} = 55.$

In wooden beams of relatively large depth compared with breadth the question of horizontal shear must be investigated. A proper analysis of horizontal shear is beyond the scope of this text, but it may be stated that for beams of rectangular cross-section the horizontal shear equals $\frac{3}{2}$ the vertical shear.

Prob. 79. In Fig. 54 find the maximum vertical shear in pounds per square inch, assuming the cross-section to be 8×10.

Prob. 80. Draw the vertical shear diagram for the beam shown in Fig. 28, and compare with the bending-moment diagram shown in Fig. 29. Is the bending moment a maximum at point where the vertical shear passes through zero?

Prob. 81. Assume data given in Fig. 54. Locate the dangerous section of the beam by use of the law that the bending moment is a maximum at the point where the vertical shear passes through zero.

Art. 27. Safe Loads—Steel Beams

Wooden beams are being used less and less in building construction, owing to the fact that they do not produce a fireproof structure.

Steel beams are made of open-hearth steel having a tensile strength of 60,000 to 70,000 lbs. per sq. in. and an elongation of 25 per cent. The steel is first cast into large ingots, which are then placed into "soaking" pits, where they are heated to a high temperature. The ingots are taken from these pits by overhead cranes and passed to the rolls, where they are formed into any desired shape. The most common shape is the I section, which gives a large moment of inertia per pound weight of beam. The various steel companies roll these beams into standard shapes and weights. Table XII gives the properties of the light and heavy I beams for various depths. On the larger sizes there are two and three intermediate weights. For properties of these sections the student is referred to the handbooks prepared by the various steel companies.

Fig. 55 shows the method of increasing the weight of various rolled shapes. For a given depth the weight of the I beam is increased by increasing the thickness of the web, which necessarily increases the width of the flange. Column 6, Table XII, gives the moment of inertia of the section about the gravity axis parallel to the flanges. This is the value to be used when the I section is used as a beam. Column 8 gives the moment of inertia about the gravity axis parallel to the web. The use of this value is discussed in the chapter on columns. Column 7 gives the *section modulus*, which, as stated before, is equal to the value of $\frac{I}{c}$. In designing steel beams to carry uniformly distributed loads the safe working stress may be taken as 16,000 lbs. per sq. in.

Consider the case of a steel I beam which has a span of

30 ft. and is to carry a uniformly distributed load of 30,000 lbs. In this case the maximum bending moment equals

$$\frac{WL}{8}=\frac{30000\times30\times12}{8}=1,350,000 \text{ in-lbs.},$$

and $S=16,000$, hence

$$\frac{I}{c}=\frac{M}{S}=\frac{1350000}{16000}=84.4.$$

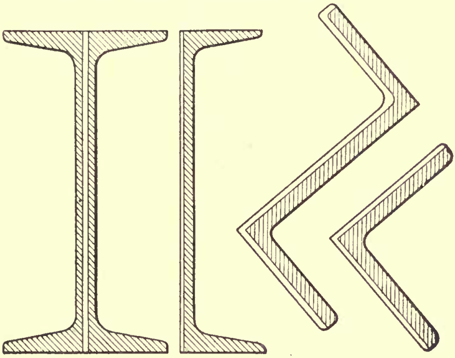


FIG. 55.

By reference to column 7, Table XII, it is found that the 18-in. beam weighing 55 lbs. per ft. comes the nearest to the desired value of $\frac{I}{c}$, and hence the 18-in. beam would be used in this case.

For beams subject to shock a fiber stress less than 16,000 must be used.

PROB. 82. A floor is to carry a load of 250 lbs. per sq. ft. and is supported by steel I beams having a span of 20 ft. The beams are spaced 4 ft. center to center. Using $S=16,000$, find the size I beam to be used.

PROB. 83. Find the safe uniformly distributed load that can be carried by a heavy 24-in. I beam having a span of 40 ft.

PROB. 84. A steel cantilever beam has a span of 10 ft., and carries a concentrated load at the end of the beam of 10,000 lbs. Find size of steel I beam to be used.

TABLE XII

1 Depth of Beam.	2 Weight per Foot.	3 Area of Section.	4 Thickness of Web.	5 Width of Flange	6 I_g Axis 1-1.	7 Section Modulus.	8 I_g Axis 2-2.	9 Radius of Gyration Axis 2-2
d Inches.	*w* Pounds.	*A* Sq. In.	*t* Inches.	*b* Inches.	*I*	*S*	*I*	*r* Inches.
3	5.50	1.63	.17	2.33	2.5	1.7	.46	.53
	7.50	2.21	.36	2.52	2.9	1.9	.60	.52
4	7.50	2.21	.19	2.66	6.0	3.0	.77	.59
	10.50	3.09	.41	2.88	7.1	3.6	1.01	.57
5	9.75	2.87	.21	3.00	12.1	4.8	1.23	.65
	14.75	4.34	.50	3.29	15.1	6.1	1.70	.63
6	12.25	3.61	.23	3.33	21.8	7.3	1.85	.72
	17.25	5.07	.47	3.57	26.2	8.7	2.36	.68
7	15.00	4.42	.25	3.66	36.2	10.4	2.67	.78
	20.00	5.88	.46	3.87	42.2	12.1	3.24	.74
8	18.00	5.33	.27	4.00	56.9	14.2	3.78	.84
	25.25	7.43	.53	4.26	68.0	17.0	4.71	.80
9	21.00	6.31	.29	4.33	84.9	18.9	5.16	.90
	35.00	10.29	.73	4.77	111.8	24.8	7.31	.84
10	25.00	7.37	.31	4.66	122.1	24.4	6.89	.97
	40.00	11.76	.75	5.10	158.7	31.7	9.50	.90
12	31.50	9.26	.35	5.00	215.8	36.0	9.50	1.01
	40.00	11.76	.56	5.21	245.9	41.0	10.95	.96
15	42.00	12.48	.41	5.50	441.8	58.9	14.62	1.08
	60.00	17.65	.75	5.84	538.6	71.8	18.17	1.01
18	55.00	15.93	.46	6.00	795.6	88.4	21.19	1.15
	70.00	20.59	.72	6.26	921.2	102.4	24.62	1.09
20	65.00	19.08	.50	6.25	1169.5	117.0	27.86	1.21
	75.00	22.06	.65	6.40	1268.8	126.9	30.25	1.17
24	80.00	23.32	.50	7.00	2087.2	173.2	42.86	1.36
	100.00	29.41	.75	7.25	2379.6	198.3	48.55	1.28

ART 28. CAST-IRON BEAMS

Owing to its low tensile strength and low ductility cast iron is not suited for beams used in building construction. It is, however, used extensively in the construction of machines where heavy bases, frames, or bedplates are required. Cast iron is readily formed into any desired shape by casting and is, therefore, the material best adapted to the purposes just stated. There are many cases where these machine parts take the nature of a beam, thus putting the member in tension. Grate bars for furnaces are made of cast iron, and these must be designed as beams.

A very common illustration of a cast-iron beam is in the case of gear teeth. The gear blank is cast and the teeth are usually cut, although on the large gears such as are used in rolling mills, etc., the teeth are cast. Fig. 56 shows a common form of tooth. Let L equal the working depth of the tooth in inches and assume that the load W is concentrated on the end of the tooth. The maximum bending moment equals WL, and the resisting moment $=\frac{SI}{c}$. Let F equal the face of the tooth in inches. The dangerous section is at A–B and the resisting moment equals

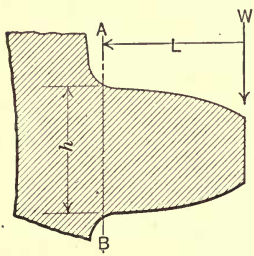


FIG. 56.

$$\frac{SFh^2}{6},$$

hence

$$WL=\frac{SFh^2}{6};$$

or

$$F=\frac{6WL}{Sh^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

but L and h are functions of the circular pitch of the teeth so that the equation for F is written

$$F=\frac{W}{Spy}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where y equals a constant, and p equals the circular pitch in inches. Mr. Wilfred Lewis has suggested proper values of y and S for cast-iron teeth. The student is referred to Kent's Handbook for further data on gear-teeth design.

EXAMPLE. In Fig. 56, $h=.87$ in., $L=1$ in., $W=2400$ lbs., and $S=3000$; find the face of tooth. Here $h=.87$; $L=1$ in. and $W=3600$. Substituting these values in Equation (30) gives

$$F=\frac{6WL}{Sh^2}=\frac{6\times2400\times1}{3000\times.87^2}=6.34 \text{ ins.}$$

The value of S depends upon the speed of the pitch line of the gear, as well as upon the material used.

Bed plates of lathes and planers are illustrations of cast-iron beams. No set rules can be given for the proportions to be used in cast-iron beams, but as a general rule it is wise to so distribute the metal as to secure the maximum section modulus with the minimum weight of metal.

PROB. 85. A cast-iron gear wheel of 24 ins. pitch diameter transmits 30 horse-power at 200 R.P.M. The teeth are 2.5 diametral pitch. Find unit working stress if the gear teeth are 5 ins. wide.

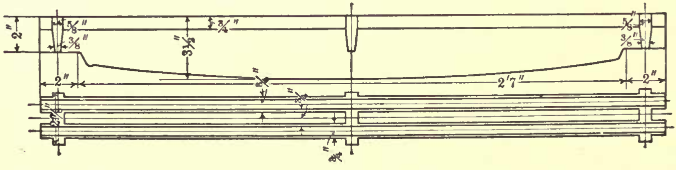


FIG. 57.

PROB. 86. A cast-iron grate bar is of the form shown in Fig. 57. If the span is 4 ft. find the safe uniformly distributed load that the bar can carry, using $S=8000$ lbs. per sq. in.

PROB. 87. Fig. 58 gives the cross-section of the bed of a lathe. The span is 6 feet. Using $S=8000$ lbs. per sq. in., find safe uniformly distributed load the bed plate can carry.

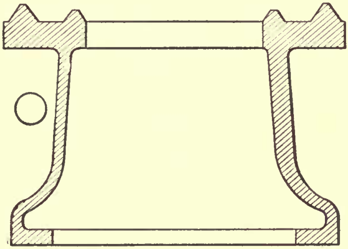


FIG. 58.

ART. 29. BEAMS OF UNIFORM STRENGTH

There are many cases where a beam of uniform strength is desirable; for example, in the case of gear teeth, but it is not always profitable to secure the beam of uniform strength at the sacrifice of other important details. In all previous examples the fiber stress has been investigated only at the dangerous section of the beam. For a beam to be of uniform strength it is necessary that the fiber stress be the same at all sections of the beam. The contour of the beam will depend upon the nature of loading. Several cases will be considered.

CASE I. Simple beam of uniform strength concentrated load at the center (see Fig. 59). Let d_1 = the depth of the beam at the center of the span and b = the breadth. Let d = the depth at any section distant x feet from the left end. From Equation (24) $S=\frac{3}{2}\frac{PL}{bd_1^2}$. At the section A–B the bending moment equals $\frac{P}{2}x$ and the resisting moment equals

$$\frac{SI}{c}=\frac{Sbd^2}{6},$$

therefore,

$$\frac{Px}{2}=\frac{Sbd^2}{6},$$

or

$$S=\frac{3Px}{bd^2}.$$

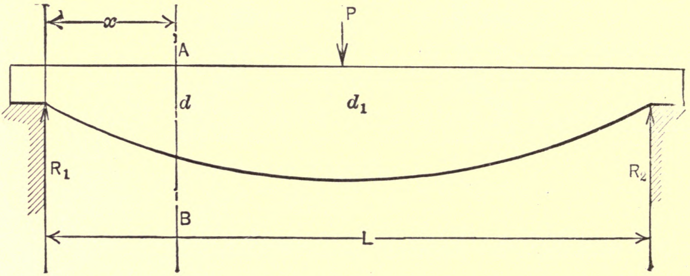


FIG. 59.

Now for the beam to be of uniform strength, the value of S at A–B must equal the value of S at the center of the span, or

$$S=\frac{3}{2}\frac{PL}{bd_1^2}=\frac{3Px}{bd^2},$$

or

$$\frac{L}{2d_1^2}=\frac{x}{d^2}.$$

Therefore

$$d=d_1\sqrt{\frac{2x}{L}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

CASE II. Simple beam. Uniformly distributed load. See Fig. 60. As before, let d_1 equal the depth at the center and d equal the depth at the section A–B distant x feet from the left end. Then, for the center section $S=\frac{3}{4}\frac{WL}{bd_1^2}$, (see

Equation (26)) at the section A–B the bending moment equals $\frac{wL}{2}x-\frac{wx^2}{2}$, and the resisting moment equals $\frac{Sbd^2}{6}$; hence

$$\frac{w}{2}(Lx-x^2)=\frac{Sbd^2}{6},$$

or

$$S=\frac{3w(Lx-x^2)}{bd^2}.$$

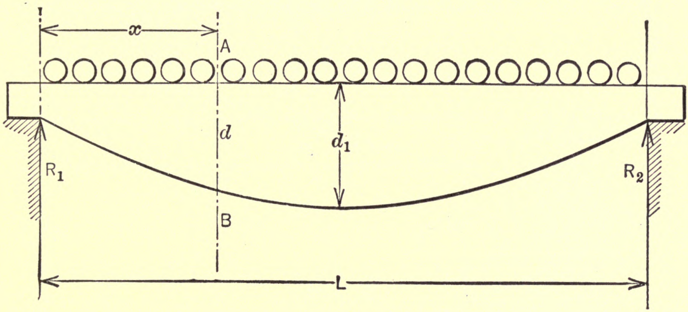


FIG. 60.

Equating these values of S gives

$$S=\frac{3}{4}\frac{wL^2}{bd^2}=\frac{3w(Lx-x^2)}{bd^2},$$

or

$$\frac{L^2}{4d_1{}^2}=\frac{Lx-x^2}{d^2},$$

or

$$d=\frac{2d_1\sqrt{x(L-x)}}{L}. \qquad (33)$$

CASE III. Cantilever beam. Concentrated load at end. See Fig. 61. Let d_1 equal the depth at the wall and d equal the depth at any section x feet from the left end.

Then at the wall section $S=\frac{6PL}{bd_1^2}$ and at section A–B the bending moment equals Px the resisting moment equals $\frac{Sbd^2}{6}$, or

$$Px=\frac{Sbd^2}{6},$$

hence

$$S=\frac{6Px}{bd^2}.$$

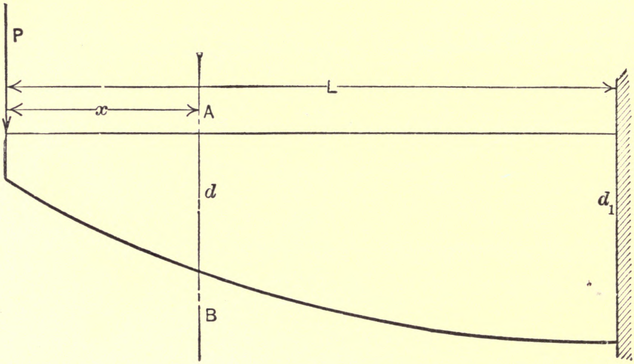


FIG. 61.

Equating these values of S gives

$$S=\frac{6PL}{bd_1^2}=\frac{6Px}{bd^2},$$

or

$$d=d_1\sqrt{\frac{x}{L}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

CASE IV. Cantilever beam. Uniformly distributed load. See Fig. 62. At the wall section $S=\frac{3wL^2}{bd_1^2}$, and at the

section $A-B$ the bending moment equals wx^2, the resisting moment equals $\frac{Sbd^2}{6}$; hence

$$S=\frac{3wx^2}{bd^2}.$$

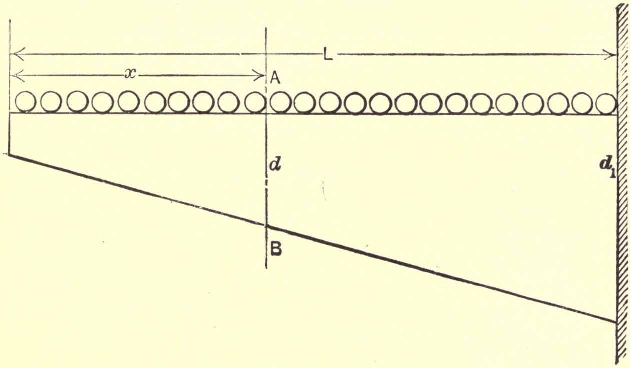


FIG. 62.

Equating these values of S gives

$$S=\frac{3wL^2}{bd_1^2}=\frac{3wx^2}{bd^2},$$

or

$$\frac{d^2}{d_1^2}=\frac{x^2}{L^2},$$

therefore

$$d=\frac{xd_1}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

PROB. 88. A simple wooden beam 6 ins. wide carries a uniformly distributed load of 5400 lbs. Design this beam so to be of uniform strength, assuming $S=1200$ and the span to be 12 ft.

Art. 30. Modulus of Rupture

Equation (16) in Art. 21 was derived on the assumption that the fiber stress varies directly with the given element of area from the neutral axis. This statement is true so long as the unit stress does not exceed the elastic limit of the material. Beyond the elastic limit the stress does not vary directly with the distance of the element from the neutral axis. Within the elastic limit the value of S will agree with the tensile or compressive strength of the material.

In testing materials under transverse tests it is customary to figure the breaking stress from the formula $S=\frac{Mc}{I}$, and to call the value thus found the "modulus of rupture" of the material. This modulus is neither equal to the tensile nor compressive strength of the material and is not to be confused with these values. For example, the tensile strength of cast iron is about 20,000 lbs. per sq. in. and the compressive strength of cast iron if 90,000 lbs. per sq. in. The "modulus of rupture" of cast iron as determined by experiment average 35,000 lbs. per sq. in.

In the case of wrought iron and mild steel there is no fixed modulus of rupture, as these materials will continue to bend under the action of a transverse load. In such cases the load carried is generally fixed by the allowable deflection. For beams a safe working stress of 16,000 can be used for mild steel.

In conducting transverse tests of wood it is essential that the span be at least ten times the depth of the beam. The ends of the specimen should rest on bearing plates rather than directly on the knife-edges of the testing machine. For any beam of rectangular cross-section tested under a concentrated load of P pounds at the center of a span of L inches, the modulus of rupture is found from the formula

$$S=\frac{3}{2}\frac{PL}{bh^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35a)$$

where P equals the load required to cause rupture of the specimen. Table XIII gives the modulus of rupture of various materials.

TABLE XIII

MODULUS OF RUPTURE

Material.	Bending Pounds Per Sq. In.	Torsion Pounds Per Sq. In.
Timber	9,000	
Cast iron	35,000	
Wrought iron	....	40,000
Copper cast	30,000	
Granite	1,900	

EXAMPLE. A 2×4-in. yellow pine beam of 42-in. span broke under a concentrated load of 5000 lbs. at the center of the span. Find the modulus of rupture. Here $P=5000$, $b=2$ ins., $h=4$ ins., $L=42$ ins., hence

$$S=\frac{3}{2}\frac{PL}{bh}=\frac{3}{2}\times\frac{5000\times42}{2\times16}=9840 \text{ lbs. per sq. in.}$$

In the case of a section which is not symmetrical with respect to the gravity axis, it is usual to assume that Equation (16) is true, and figure values of S for both top and bottom fibers. For example, in the case of the cast-iron T section shown in Fig. 63. This section was tested as a beam of 12-in. span, and broke under a load of 2400 lbs. The bottom fibers were in tension and the top fibers were in compression. The maximum fiber stress in tension is found from the equation $S=\frac{Mc}{I}$, using for c the distance from the neutral axis to the base of the beam, which in this case is .46 in. The bending moment

$$M=\frac{PL}{4}=\frac{2400\times12}{4}=7200 \text{ in.-lbs.}$$

The moment of inertia relative to the gravity axis is approximately .08. Substituting these values in the above equation gives

$$S=\frac{7200\times .46}{.08}=41{,}400 \text{ lbs per sq. in.}$$

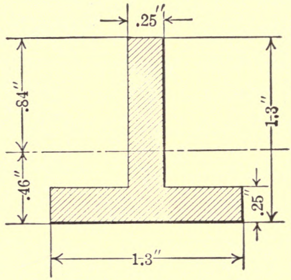


FIG. 63.

PROB. 89. A piece of white pine 2×4 ins. cross-section and 42 ins. span broke under a transverse concentrated load of 5000 pounds. Find the modulus of rupture.

PROB. 90. A cast-iron T section of form shown in Fig. 63, having the following dimensions, length flange = 1.25 ins., depth of section = 1.25 ins., thickness of we band flange = .30 ins., span 12 ins. broke under a load of 2200 lbs. Find the maximum fiber stress in tension and compression. Load applied on top of web, flange horizontal.

ART. 31. PRACTICAL PROBLEMS

PROB. 91. A floor carrying a load of 100 lbs. per sq. ft., is supported by 3×8-in. yellow-pine beams of 12-ft. span. Find proper spacing of beams.

PROB. 92. The beams in Prob. 91 are supported by wooden girders of 24-ft. span. The depth of the girders is twice the breadth. Find the dimensions of the girders.

PROB. 93. A floor is supported by steel I beams, of 20-ft. span

and 6-ft. centers. The floor sustains a load of 200 lbs. per sq. ft. Find size I beams required.

PROB. 94. A yellow-pine beam is loaded as shown in Fig. 64. Find the maximum fiber stress and the factor of safety.

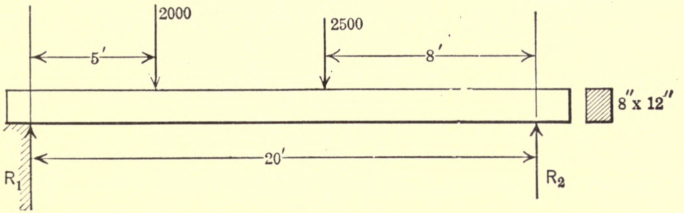


FIG. 64.

PROB. 95. Fig. 65 shows the arrangement of a pin connection in a bridge truss. Find diameter of the pin, using $S = 8000$ lbs. per sq. in.

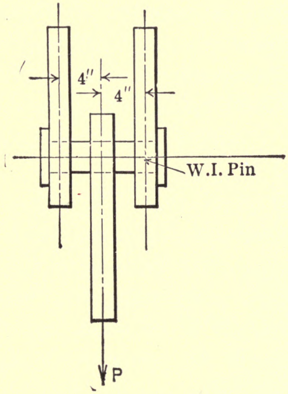


FIG. 65.

PROB. 96. A 12-in. steel I beam of 30-ft. span supports a load of 500 lbs. per lin. ft. Is the beam safe?

PROB. 97. A traveling crane consists of two heavy 18-in. I beams of 40-ft. span. Neglecting the weight of the trolley and

hoist, find maximum load that can be raised so that the safe fiber stress shall not exceed 12,000 lbs. per sq. in.

PROB. 98. A cast-iron beam $1\frac{1}{2}\times2\frac{1}{2}$ ins. in section and 30 ins. span broke under a concentrated load of 7000 lbs. Find the modulus of rupture.

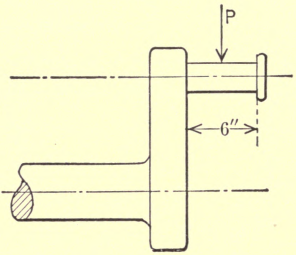


FIG. 66.

PROB. 99. Fig. 66 shows the crank pin of a side crank engine. Find the proper diameter of steel pin when the load $P=24{,}000$ lbs. Find the factor of safety in shear.

PROB. 100. Compare the safe loads that can be carried by a yellow-pine beam 8×10 ins. in cross-section and 20-ft. span, and a light 10-in. I beam of 30-ft. span.

CHAPTER VI

DEFORMATIONS

Art. 33. Modulus of Elasticity

The term deformation is used to designate the change of form of any part of a machine or structure when subjected to an external load. As stated previously in the case of tension the deformation becomes an elongation, in the case of compression the deformation is a shortening of the part; in the case of bending the deformation takes the form of a deflection of the neutral axis of the part; and in the case of torsion the deformation becomes a twisting action of the external fibers of the part. The relation between the external load applied and the resulting deformation is not always a simple one to determine, but in general this relation can be expressed as a ratio, which is referred to as the "modulus of elasticity."

The *modulus of elasticity* is equal to the ratio of the unit stress to the unit deformation at the elastic limit of the material. So long as the load does not exceed the elastic limit, this ratio is assumed to be constant. However, under actual test the ratio is found to vary for the lighter loads. Let E equal the modulus of elasticity, S equal the unit stress in pounds per square inch, and d equal the unit deformation, then

$$E=\frac{S}{d}, \qquad (36)$$

or

$$d=\frac{S}{E}. \qquad (37)$$

In the case of a tension test let P equal the load at the elastic limit, L equal the length in inches of the part of the specimen under test, e equal the total elongation at the elastic limit and A equal the original cross-section of the specimen; then

$$S=\frac{P}{A},$$

and

$$d=\frac{e}{L},$$

or

$$E=\frac{P}{A}\cdot\frac{L}{e}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (38)$$

For example, a specimen of the form shown in Fig. 12 gave the following results in a tension test; load at elastic limit was 7000 lbs., elongation at elastic limit .0024 in.

The unit stress at the elastic limit equals $\frac{P}{A}=\frac{7000}{.196}=35,800.$

and

$$\text{the unit deformation equals } \frac{.0024}{2}= 0012 \text{ in.};$$

hence

$$E=\frac{S}{d}=\frac{35800}{.0012}=30,000,000 \text{ (approx.)}.$$

The modulus of elasticity is an index of the stiffness of the material, so that if any machine part is under tension and the unit stress does not exceed the elastic limit of the material, the probable elongation of the part can be figured if the modulus of elasticity is known.

For example, consider the case of a steel tension member to be $1\frac{1}{2}\times 4$ ins. cross-section and 10 ft. long, and carrying a load of 72,000 lbs. The unit stress in this case equals $\frac{72000}{6}=12,000$ lbs. per sq. in. The total elongation which

will probably occur may be found by arranging Equation (38) in the form

$$e=\frac{PL}{AE}, \qquad \dots\dots\dots \qquad (39)$$

thus

$$e=\frac{72000\times 120}{6\times 30000000}=.048 \text{ in.}$$

In the design of any structure proper allowance must be made for the increase in length of the various members when subjected to a tensile stress.

In like manner the probable shortening can be determined in the case of a compression test. In this case the value of e in Equation (38) is to be taken as the decrease in length of the specimen, and the other terms are the same as in tension. However, this equation holds true only when the specimen is under pure compression, and must not be confused with the case where the specimen acts as a column. (See Chapter VIII.)

The values of the modulus of elasticity are given in Table XIV. It will be noted that the modulus is about the same in tension and compression. In the case of torsion the term modulus of rigidity is sometimes used.

TABLE XIV

Material.	Modulus of Elasticity.		
	Tension.	Compression.	Shear.
White pine	1,130,000	1,130,000	
Yellow pine	1,500,000	1,500,000	
White oak	1,150,000	1,150,000	
Cast iron	12,000,000	12,000,000	
Wrought iron	28,000,000	28,000,000	
Copper wire	18,000,000	18,000,000	
Steel (plate)	29,000,000	29,000,000	
Steel (machine)	30,000,000	30,000,000	

PROB. 101. A steel specimen 0.8 in. in diameter and 8 ins. long was subjected to a load of 18,000 lbs. with a resulting elongation of .0096. Find the modulus of elasticity.

PROB. 102. How much will a wrought-iron bar 1×2 in. and 6 ins. long shorten under a load of 50,000 lbs.?

ART. 33. DEFLECTION OF BEAMS

The relation between the unit stress and the unit deformation in the case of beams is not as easily determined as in the case of tension and compression. When a beam is subjected to a bending moment the lower fibers are put in tension and the bottom part of the beam is elongated, while the top part is shortened. The neutral axis remains the same length. Thus, in Fig. 67 *ox* represents the position of the neutral

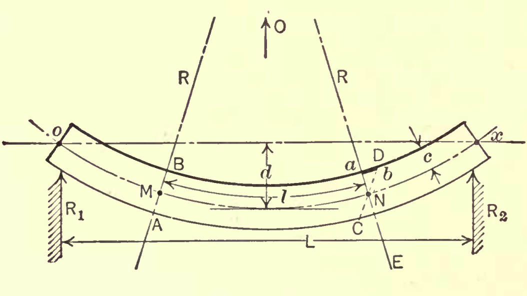


FIG. 67.

axis when the external bending moment is zero and the line *oMNx* the position of the neutral axis under the action of the external bending moment. At points *M* and *N* construct the normals *OA* and *OE*. These normals intersect at some point *O*, which is the center of curvature for the line *MN*. The distance *ab* represents the deformation for the element of length represented by *MN* or *l*. The deformation per unit of length equals $\frac{ab}{l}$, hence the modulus of elasticity equals the unit stress divided by the unit deforma-

tion, or $E=\frac{Sl}{ab}$, or a $b=\frac{SL}{E}$. If l is taken as a very short distance, it will be practically a straight line, and then the triangle OMN and the triangle Nab are similar, hence $\frac{R}{l}=\frac{c}{ab}$, but $ab=\frac{SL}{E}$, therefore $\frac{R}{l}=\frac{c \cdot E}{SL}$ or $\frac{R}{c}=\frac{E}{S}$. The unit stress $S=\frac{Mc}{I}$; substituting this value of S in the above equation gives

$$\frac{R}{c}=\frac{EI}{Mc},$$

or

$$M=\frac{EI}{R}. \qquad (40)$$

Equation (40) is referred to as the equation of the elastic curve. The value of R will depend upon the nature of loading of the beam. Its value can be computed only by the aid of the calculus. It can be seen, however, that the deflection d of the beam will depend upon the radius of curvature R of the elastic curve.

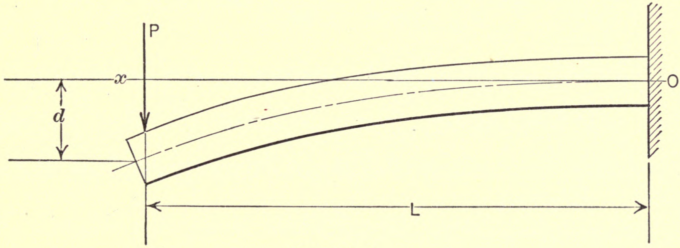


FIG. 68.

In the case of a cantilever beam with a load of P pounds at the end of a beam of L inches length, the maximum deflection will be directly under the load P as shown in Fig. 68. The value of this deflection is given by the formula

$$d=\frac{PL^3}{3EI}. \qquad (41)$$

where P = the load in pounds;
L = the length in inches;
E = the modulus of elasticity;
I = the moment of inertia of the section, and
d = the maximum deflection in inches.

As an application of the formula consider the case of a 6×9 ins. yellow-pine cantilever beam, which extends 6 ft. beyond the support and carries a concentrated load of 1350 lbs. The maximum deflection in this case equals

$$\frac{PL^3}{3EI}.$$

Where $P=1350$;
$L^3=72^3=373{,}248$;
$E=1{,}500{,}000$;

and

$$I=\frac{bd^3}{12}=\frac{6\times729}{12}=364.5;$$

hence

$$d=\frac{1350\times373248}{3\times1500000\times364.5}=0.3\text{ in.}$$

Equation (41) can also be written in terms of the maximum unit fiber stress. From Equation (16) $S=\frac{Mc}{I}$, and in the case of a cantilever beam with concentrated load at the end $M=PL$, hence

$$S=\frac{Mc}{I}=\frac{PLc}{I},$$

or

$$P=\frac{SI}{Lc}.$$

Substituting this value in Equation (41) gives

$$d=\frac{SI}{Lc}\cdot\frac{L^3}{3EI}=\frac{SL^2}{3Ec}. \qquad (42)$$

From this equation it is seen that for a given fiber stress the deflection of a beam varies directly as the square of the length.

For a cantilever beam with a uniformly distributed load of W pounds the deflection is given by the formula,

$$d=\frac{WL^3}{8EI}. \quad \ldots\ldots\ldots \quad (43)$$

Here again, the deflection may be expressed in terms of the unit stress for

$$S=\frac{Mc}{I};$$

but

$$M=\frac{WL}{2};$$

hence

$$S=\frac{WL}{2}\cdot\frac{c}{I}=\frac{WLc}{2I},$$

or

$$W=\frac{2SI}{Lc}.$$

Substituting this value in Equation (43) gives

$$d=\frac{2SIL^3}{8LcEI}=\frac{SL^2}{4Ec}. \quad \ldots\ldots \quad (44)$$

For a simple beam with a concentrated load of P pounds at the center of the span the maximum deflection is given by the equation

$$d=\frac{1}{48}\frac{PL^3}{EI}. \quad \ldots\ldots\ldots \quad (45)$$

or in a similar manner to the above it can be shown that

$$d=\frac{SL^2}{12Ec} \quad \ldots\ldots\ldots\ldots \quad (46)$$

For a simple beam with a uniformly distributed load of W pounds the deflection is given by the equation

$$d=\frac{5PL^3}{384EI}. \qquad (47)$$

or

$$d=\frac{5SL^2}{48Ec}. \qquad (48)$$

PROB. 103. Find the maximum deflection of a 12-in. heavy steel I beam of 25-ft. span which carries a uniformly distributed load of 10,000 lbs.

PROB. 104. A steel shaft 6 ins. diameter rests on two bearings which are 6 ft. apart. The shaft carries a flywheel weighing 2400 lbs. located at the center of the span. Find the deflection due to the weight of the flywheel.

ART. 34. PROBLEMS

PROB. 105. Assume the diameter of the crank pin in Fig. 66 equals 4 ins. The maximum allowable deflection at the end of the pin is .01 in. Find the load P required to give this deflection.

PROB. 106. A cast-iron water pipe 24 ins. in diameter rests on supports which are located 20 ft. apart. Find the deflection of the pipe when filled with water. Thickness of the pipe equals $1\frac{1}{8}$ ins.

PROB. 107. A 15-in. heavy steel I beam of 30-ft. span carries a uniformly distributed load. Find the value of this load so that the maximum deflection shall not exceed $\frac{1}{360}$ of the span.

PROB. 108. A floor is supported by 3×8 wooden beams spaced 18 ins. apart. Find the load the floor can carry in pounds per square foot, so that the deflection of the ceiling supported under the floor will not exceed the allowable value of $\frac{1}{360}$ of the span.

PROB. 109. Two beams of uniform cross-section and having span of 10 ft. and 20 ft. respectively carry concentrated loads producing equal fiber stresses in both beams. Find the relative deflection of the two beams.

PROB. 110. In a transverse test of a simple wood beam of 42-in. span the load at the elastic limit was 2400 lbs., and the cor-

responding deflection 0.154 in. Find the modulus of elasticity. Beam was 3 ins. in breadth and 4 ins. in depth. Load was applied at the center of the span.

PROB. 111. Determine the spacing of light 10-in. I beams which carry a uniform floor load of 100 lbs. per sq. ft., so that the maximum deflection shall not exceed $\frac{1}{360}$ of the span. The beams are 18-ft. span.

PROB. 112. In Fig. 48 find the maximum deflection when the load P equals 3000 lbs.

PROB. 113. A box girder of the form shown in Fig. 49 has a span 30 ft. Find the uniformly distributed load required to produce a maximum deflection of 0.75 in. at the center of the span. Plates are 14 ins. wide.

PROB. 114. In a transverse test of a cast-iron beam 1×1 in. in section and 12 ins. span the load at the elastic limit was 2800 lbs., and the deflection 0.08 in. Find the modulus of elasticity of cast iron. Load applied at the center of the span.

CHAPTER VII

SHAFTING

Art. 35. Twisting Moment

When an external load is applied tangential to the circumference of a pulley there is produced a tendency to distort the shaft on which the pulley is mounted; that is, the neutral axis of the shaft will not be affected, but the outer fibers of the shaft will tend to take the form of spirals. Under such a condition the shaft is said to be under *torsion*, and the stress produced in the shaft is one of *shear*. The product of the tangential force times the radius of the pulley is called the twisting moment, which moment may be negative or positive.

The amount of distortion or twist of the outer fiber of the shaft is proportional to the external load applied, provided the elastic limit of the material is not exceeded.

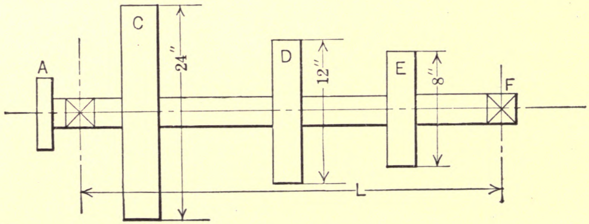


Fig. 69.

The twisting moment at various sections of a shaft is not necessarily the same. For example, in Fig. 69 let *C*, *D*, and *E* be pulleys mounted upon a shaft to which power is delivered

by the coupling A. Assume that belts are placed on the pulleys and that the belt pulls are 200, 150, and 100 lbs. on the pulleys C, D, and E, respectively, and that the belts are traveling in the same direction. The twisting moment on the section between the pulley E and the bearing F will equal zero; between the pulleys D and E the twisting moment equals $4 \times 100 = 400$ in.-lbs. Between pulleys C and D the twisting moment equals $6 \times 150 + 4 \times 100 = 1300$ in.-lbs. Between pulley C and the coupling A the twisting moment equals $12 \times 200 + 150 \times 6 + 4 \times 100 = 3700$ in.-lbs.

The twisting moment on a shaft is sometimes referred to as the torque. For instance, in the case of a motor shaft, the magnetic pull of the field tends to rotate the armature of the motor, which, in turn, produces a twisting moment or torque on the shaft.

Test specimens are tested in torsion by placing the specimen in jaws which in turn are placed in the heads of a testing machine. One of these heads is fixed and the other is slowly rotated, setting up a *torque* or *twist* in the specimen. The angular deformation is measured by the number of degrees through which the movable head is turned.

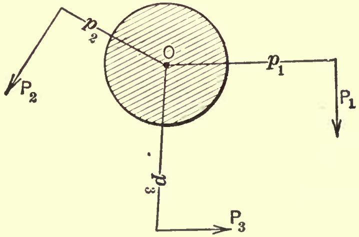


FIG. 70.

Twisting moments like bending moments should be figured in inch-pounds. In Fig. 70, let P_1, P_2 and P_3 represent forces acting at distances of p_1, p_2, p_3 from the center of the shaft. The force P_1 tends to rotate the shaft in a clock-

wise direction, and produces a positive twisting moment. The forces P_2 and P_3 tend to rotate the shaft counter-clockwise, and produce negative twisting moments. The resultant twisting moment is the algebraic sum of the moments of all the forces tending to twist or rotate the shaft. In this case the resultant twisting moment equals $+P_1p_1-P_2p_2-P_3p_3$, or, expressed as an equation,

$$T=\Sigma Pp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

It is to be noted that in figuring twisting moments the moment arm is taken as the perpendicular distance from the center of the shaft to the line of action of the force.

PROB. 115. A pulley 48 ins. in diameter is placed on the end of a shaft. A belt having an effective belt pull of 250 lbs. is placed on the pulley. Find the twisting moment produced in the shaft.

ART. 36. RESISTING MOMENT IN TORSION

A twisting moment produces a shear in the cross-section of a shaft which varies from 0 at the neutral axis to a maximum at the outer fiber or circumference of the shaft. Within the elastic limit of the material this unit shear is proportional to the distance of the fiber from the neutral axis. Let Fig. 71 represent the cross-section of a shaft at which point the external twisting moment equals $P\times p$. Let $S=$ the fiber stress on the outermost fiber of the shaft and let a equal an element of the cross-section of the shaft located at a distance of z units from the neutral axis, which passes through the point O. Assuming that the elastic limit of the material is not exceeded, the unit stress on the element of area a equals $S\frac{z}{c}$, where c is the distance from the neutral axis to the outermost fiber. The total stress on this small element of area equals $S\frac{z}{c}\cdot a$. As the external twisting moment tends to

shear the shaft, the internal stress on area a tends to exert an *internal resisting* moment, which moment equals the stress $S\frac{z}{c}\cdot a$, times its moment arm z, or the resisting moment of the small element of area equals $S\frac{z}{c}\cdot a\cdot z=\frac{S}{c}az^2$. Now consider the cross-section of the shaft to be made up of N areas of the form a, and let A equal the cross-sectional

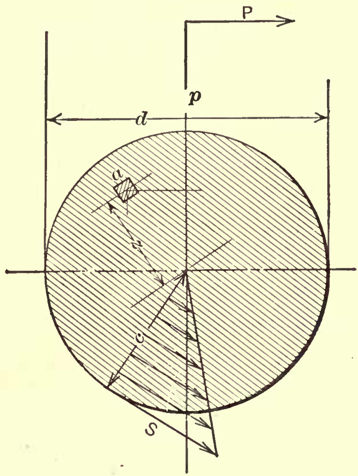


FIG. 71.

area of the shaft. Then $A=aN$. The total resisting moment of the entire cross-section equals the sum of the resisting moments of the elementary areas. Let T equal this resisting moment. Then

$$T=\frac{S}{c}(az_1{}^2+az_2{}^2+az_3{}^2+az_n{}^2),$$

$$=\frac{Sa}{c}[z_1{}^2+z_2{}^2+z_3{}^2+z_n{}^2].$$

Let $Z^2=$ the average value of the terms $z_1{}^2$, $z_2{}^2$, $z_3{}^2$, etc.

Then $z_1^2+z_2^2+z_3^2+z_4^2+=Z^2N.$

or

$$T=\frac{Sa}{c}[NZ^2],$$

but

$$aN=A,$$

hence

$$T=\frac{S}{c}[AZ^2].$$

The expression AZ^2 is called the *polar moment of inertia* of the section. Let J equal the polar moment of inertia. For equilibrium to exist the external twisting moment must equal the internal resisting moment, hence

$$Pp=\frac{SJ}{c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

Equation (50) is similar to Equation (15), and as Equation (15) is fundamental in the design of beams, so Equation (50) becomes fundamental for the design of shafts subjected to torsion only.

PROB. 116. A shaft 6 ins. in diameter is subjected to an external twisting moment such that the maximum shear on the outermost fiber is 3000 lbs. per sq. in. What is the average shear on each square inch of cross-section?

ART. 37. POLAR MOMENT OF INERTIA

The polar moment of inertia of any section is equal to the sum of the products obtained by multiplying each element of area by the square of its distance from the center of the section. The term polar is used because the distances are measured from a point or *pole*. In the design of beams the moment of inertia was obtained by multiplying each element of area by the square of its distance from a given axis. This moment of inertia is called the rectangular mo-

ment of inertia. The polar moment of inertia of a section equals the sum of the rectangular moments of inertia taken about axes which pass through the center of the section and are perpendicular to each other. In Fig. 72 let a equal an element of area located at a distance of z units from the center of the surface, and at a distance of x units from the axis OY and a distance of y units from the axis OX. Then by definition the rectangular moment of inertia relative to the OY axis equals $Iy=\Sigma ax^2$. In like manner the rectangular moment of inertia about the OX axis equals $I_x=\Sigma ay^2$. Now, by definition the polar moment of inertia of the section

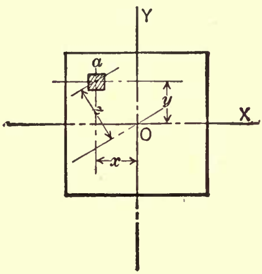


FIG. 72.

relative to the center O equals $J=\Sigma az^2$. By inspection of the figure it is evident that $z^2=x^2=y^2$; hence

$$\Sigma az^2=\Sigma ax^2+\Sigma ay^2, \quad \text{or} \quad J=I_x+I_y. \quad . \quad . \quad (51)$$

Equation (51) gives a simple method for finding the polar moment of inertia when the rectangular moment is known. Thus, in the case of a circle of diameter d_1 the rectangular moment of inertia equals

$$I_x=\frac{\pi d^4}{64};$$

also

$$I_y=\frac{\pi d^4}{64};$$

hence

$$J = I_x + I_y = \frac{\pi d^4}{64} + \frac{\pi d^4}{64} = \frac{\pi d^4}{32}. \quad . \quad . \quad . \quad (52)$$

For sections which are symmetrical with respect to the x and y axes, the polar moment of inertia equals twice the rectangular moment.

For the hollow circular section shown in Fig. 73 the polar moment of inertia,

$$J = \frac{\pi}{32}(d_1^4 - d^4). \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

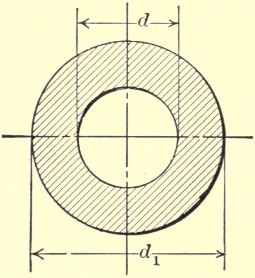


Fig. 73.

Prob. 117. Find the polar moment of inertia of a square in terms of the length of one side. Also find polar moment for a hollow square.

Prob. 118. Find the polar moment of inertia of a hollow steel shaft 12 ins. in outside diameter and 9 ins. inside diameter.

Prob. 119. Find the diameter of a solid shaft whose cross-section has a polar moment of inertia equal to that of the shaft in Prob. 118. Compare the relative weights of the shafts.

Art. 38. Shafts in Pure Torsion

When a shaft is subjected to torsion only, the internal stress in the shaft is one of shear, and the maximum fiber stress in shear is given by the equation,

$$S=\frac{Tc}{J}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (54)$$

where T equals the maximum twisting moment in inch-pounds, c equals the distance from the center of the shaft in the outermost fiber and J equals the polar moment of inertia of the section. It is more convenient to express the unit stress in terms of the horse-power transmitted and the speed of the shaft. To derive this relation several definitions are necessary.

Work is the overcoming of an external resistance through a given space. The unit of work is the foot-pound, which is the overcoming of a resistance of 1 lb. through a distance of 1 ft.

Power is the time rate of doing work, and the common unit of power is the horse-power, which is the equivalent of 33,000 ft.-lbs. of work per minute. For example, a weight of 5 tons is hoisted at the rate of 150 ft. per minute. Here the horse-power developed equals $\frac{5\times 2000\times 150}{33000}=27.3$. In Fig. 71 assume that a shaft d inches in diameter is being rotated at the rate of N revolutions per minute by a force of P pounds, acting at a distance of p feet from the center of the shaft. The work done per revolution equals $2\pi p\cdot P$ ft.-lbs. since the force P moves through a distance equal to the circumference of a circle of radius p. The work done per minute equals $2\pi p\cdot PN$ ft.-lbs., hence the horse-power,

$$H=\frac{2\pi pPN}{33000}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

or

$$Pp=\frac{33000\times H}{2\pi N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

where Pp equals the twisting moment in foot-pounds. If the twisting moment is expressed in inch-pounds, Equation (56) becomes

$$Pp=\frac{12\times 33000\times H}{2\pi N}=\frac{63025H}{N}, \quad . \quad . \quad . \quad (57)$$

but from Equation (50) $Pp=\frac{SJ}{c}$,

hence

$$\frac{SJ}{c}=\frac{63025H}{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

For a solid shaft of diameter d inches, $J=\frac{\pi d^4}{32}$ and $c=\frac{d}{2}$; substituting these values in Equation (58) gives

$$S\times\frac{\frac{\pi d^4}{32}}{\frac{d}{2}}=\frac{63025H}{N},$$

or

$$Sd^3=\frac{321000H}{N}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

or

$$H=\frac{Sd^3N}{321000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

For example, a 2-in. steel shaft running at 225 R.P.M. will transmit 33.6 horse-power with a fiber stress of C000 lbs. per sq. in.

For

$$H=\frac{6000\times 2^3\times 225}{321000}=33.6.$$

The value to be used for S will depend upon the speed of rotation. As a general rule at the higher speeds of rotation a higher factor of safety is desirable.

As an example in design let it be required to find the diameter of a solid steel shaft to transmit 100 horse-power at 200 R.P.M., using the safe working stress as 6000 lbs. per sq. in. Here $H=100$, $N=200$, and $S=6000$. Substituting these values in Equation (59) gives

$$6000\times d^3=\frac{321000\times 100}{200},$$

or

$$d^3=\frac{321000\times 100}{6000\times 200}=27,$$

hence

$$d=\sqrt[3]{27}=3.$$

For a hollow shaft of outside diameter d_1, and inside diameter d, the polar moment of inertia

$$J=\frac{\pi}{32}(d_1^4-d^4),$$

and

$$c=\frac{d}{2},$$

hence Equation (58) becomes

$$\frac{S\times\frac{\pi}{32}(d_1^4-d^4)}{\frac{d_1}{2}}=\frac{63025H}{N},$$

or

$$\frac{S(d_1^4-d^4)}{d_1}=\frac{321000H}{N}. \quad . \quad . \quad . \quad (61)$$

and

$$H=\frac{S(d_1^4-d^4)N}{321000d_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

For example, consider the case of the tail shaft of a marine engine. Inside diameter of shaft is 9 ins. and outside diameter is 12 ins. R.P.M.$=90$ and $S=7500$ lbs. per sq. in. The

horse-power transmitted is found by substituting these values in Equation (62). Thus,

$$H=\frac{7500\times(12^4-9^4)\times 90}{321000\times 12}=2484.$$

PROB. 120. A line shaft $2\frac{1}{2}$ ins. diameter runs at 225 R.P.M. What horse-power can be transmitted with a fiber stress of 7000 lbs. per sq. in.?

PROB. 121. Compare the horse-power transmitted by solid and hollow shafts of equal areas Diameter solid shaft is 8 ins. and outside diameter of hollow shaft is 12 ins. Assume shafts run at same speed and fiber stress is same in each case.

ART. 39. SHAFTS FOR COMBINED TORSION AND BENDING

In many cases a shaft is subjected to both a bending moment and a twisting moment. For example, the crank shaft of an engine is subjected to a twisting moment due to the thrust of the connecting rod on the crank pin, and also to a bending moment due to the weight of the flywheel, or the revolving element of a generator. In such cases it is necessary to figure the maximum bending moment and the maximum twisting moment and then determine the equivalent bending or twisting moment which will produce the same effect as the combined action of the bending and twisting moments.

Let M equal the maximum bending moment in inch-pounds and T equal the maximum twisting moment. Also let Te equal the equivalent twisting moment in inch-pounds due to the combined action of the bending moment M, and the twisting moment T; and Me equal the equivalent bending moment in inch-pounds.

Then

$$Me=\frac{M}{2}+\frac{1}{2}\sqrt{M^2+T^2}, \quad . \quad . \quad . \quad . \quad (63)$$

and

$$Te=\sqrt{M^2+T^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

From Equation (15) $M=\frac{SI}{c}$ and from Equation (50) $T=\frac{SJ}{c}$. For a solid shaft these formulæ become

$$M=\frac{Sd^3}{10.2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

and

$$T=\frac{Sd^3}{5.1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

When the shaft is subjected to both twisting and bending the equivalent moments are to be used, and the above equations become

$$Me=\frac{Sd^3}{10.2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

or

$$d=\sqrt[3]{\frac{10.2Me}{S}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

and

$$Te=\frac{Sd^3}{5.1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

or

$$d=\sqrt[3]{\frac{5.1Te}{S}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

In a given problem the diameter of shaft is to be figured from Equations (68) and (70), and the larger value to be taken as the required diameter of shaft.

Example. Consider the case of a crank shaft where the twisting moment is 300,000 in.-lbs. and the bending moment 240,000 in.-lbs. Let it be required to find the diameter of a solid shaft so that the safe working stress in tension shall be 10,000 lbs. per sq. in. and in shear 8000 lbs. per sq. in. In this case $M=240,000$ and $T=300,000$. Substituting these

values in Equation (63) gives as an equivalent bending moment,

$$Me = \frac{240000}{2} + \frac{1}{2}\sqrt{240000^2 + 300000^2};$$

$$= 120000 + 192000 = 312000 \text{ in.-lbs.}$$

The diameter of shaft is found from Equation (68) thus:

$$d = \sqrt[3]{\frac{10.2 \times 312000}{10000}} = 6.82 \text{ in.}$$

The equivalent twisting moment is found from Equation (64) or

$$Te = \sqrt{240000^2 + 300000^2} = 384000 \text{ in.-lbs.}$$

Substituting this value in Equation (70) gives

$$d = \sqrt[3]{\frac{5.1 \times 300000}{8000}} = 5.76 \text{ ins.}$$

This shows that the diameter of shaft required is the larger value, or 6.82 ins.

No attempt has been made to derive Equations (64) and (63), as the theory involved is almost too complicated for a text of this character. For line shafts carrying gears or pulleys a number of empirical formulæ have been suggested. Table XV gives values recommended by the A. & F. Brown Co., of New York. It will be noted that the commercial size of shafting is given in sixteenths, and advances by $\frac{1}{4}$-in. increments. The fiber stress used in this table is about 3500 to 4000 lbs. per sq. in.

TABLE XV

Horse-power of Shafts

When shafts are used for conveying power from one point to another without any of the bending strains of pulleys, gears, etc., the next smaller size may be used.

This table must not be confounded with tables of actual strength of shafts published by other authorities.

Horse-power of shafts for given diameter and speed.

Diameter of Shaft.	Revolutions per Minute.									
	100	125	150	175	200	225	250	300	350	400
$1\frac{3}{16}$ in.	2.4	3.0	3.6	4.2	4.8	5.4	6.0	7.2	8.4	9.6
$1\frac{7}{16}$ in.	4.3	5.4	6.5	7.6	8.6	9.8	10.8	13.0	15.2	17.2
$1\frac{11}{16}$ in.	6.5	8.0	9.7	11.2	13.0	14.6	16.0	19.4	22.4	26.0
$1\frac{15}{16}$ in.	10.0	12.5	15.0	17.5	20.0	22.5	25.0	30.0	35.0	40.0
$2\frac{3}{16}$ in.	14.0	17.8	21.0	24.5	28.0	31.5	35.6	42.0	49.0	56.0
$2\frac{7}{16}$ in.	20.0	25.0	30.0	35.0	40.0	45.0	50.0	60.0	70.0	80.0
$2\frac{11}{16}$ in.	26.5	32.5	40.0	44.6	53.0	59.0	65.0	80.0	89.0	106.0
$2\frac{15}{16}$ in.	34.0	42.5	51.0	59.5	68.0	76.5	85.0	102.0	119.0	136.0
$3\frac{7}{16}$ in.	54.0	67.5	81.0	94.5	108.0	122.0	135.0	162.0	189.0	216.0
$3\frac{15}{16}$ in.	80.0	100.0	120.0	140.0	160.0	180.0	200.0	240.0	280.0	320.0
$4\frac{7}{16}$ in.	114.0	142.5	171.0	199.5	228.0	256.5	285.0	342.0	399.0	456.0
$4\frac{15}{16}$ in.	156.0	195.0	234.0	273.0	312.0	351.0	390.0	468.0	546.0	624.0
$5\frac{3}{8}$ in.	208.0	260.0	312.0	364.0	416.0	468.0	520.0	624.0	728.0	832.0
$5\frac{7}{8}$ in.	270.0	337.5	405.0	472.5	540.0	607.5	675.0	810.0	945.0	1080.0
$6\frac{3}{8}$ in.	340.0	425.0	510.0	595.0	680.0	765.0	850.0	1020.0	1190.0	1360.0
$6\frac{7}{8}$ in.	420.0	525.0	630.0	735.0	840.0	945.0	1050.0	1260.0	1470.0	1680.0
8 in.	640.0	800.0	960.0	1120.0	1280.0	1440.0	1600.0	1920.0	2240.0	2560.0

It is well to say, in this connection, that no matter what general rules are adopted, there are frequently special cases in which the engineer or designer must depart from his rules, and use his judgment in determining both the size of the shaft and the number and location of bearings.

PROB. 122. The maximum twisting moment in a crank shaft is 150,000 in.-lbs. and the bending moment is 120,000 in.-lbs. Find the proper diameter of shaft, using a safe shearing stress of 6000 lbs per sq. in. and a bending stress of 8000 lbs. per sq. in.

ART. 40. PRACTICAL PROBLEMS

PROB. 123. Fig. 74 shows the arrangement of the crank shaft for a vertical side crank engine driving a direct-connected generator.

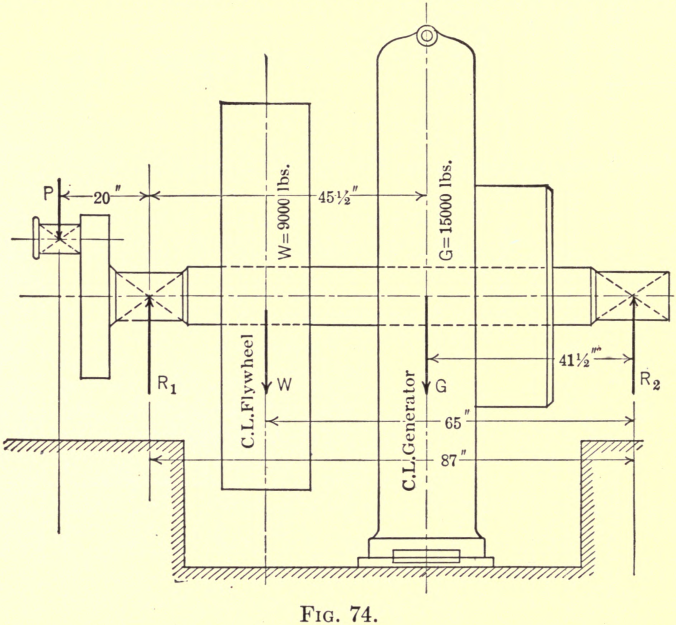


FIG. 74.

Diameter of cylinder is 24 ins. and stroke is 18 ins. Maximum unbalanced steam pressure is 100 lbs. per square in. Find the reactions R_1 and R_2 for up and down strokes.

PROB. 124. Construct the bending moment diagrams for both up and down strokes, and determine the maximum bending moment.

PROB. 125. Draw the vertical shear diagrams for both up and down strokes.

PROB. 126. Determine the maximum twisting moment, assuming that the maximum tangential thrust on the crank is equal to the maximum thrust on the piston.

PROB. 127. Determine the equivalent bending and twisting moments in inch-pounds.

PROB. 128. Using S as 10,000 lbs. per sq. in. in tension and as 8000 in shear, determine proper diameter of shaft.

PROB. 129. Using the diameter found in Prob. 128 figure the probable weight of the shaft.

PROB. 130. With the assumed diameter figure the maximum fiber stress, taking into account the weight of the shaft.

PROB. 131. Determine the diameter of the crank pin, assuming that the length of the pin equals $1\frac{1}{2}$ times the diameter.

CHAPTER VIII

COLUMNS

Art. 41. Stresses in Columns

When a short specimen is placed under a compressive load, the specimen tends to fail in shear. For example, a cast-iron specimen of the form shown in Fig. 8 will fail by shearing along the plane AB. This plane makes an angle of 45° plus the angle of repose of the material, with the vertical. As the length of the specimen relative to the diameter is increased, this angle of shear becomes less marked, and eventually the specimen will fail by buckling of the fibers. A specimen which has a length of at least ten times the least cross-sectional dimension of the specimen is referred to as a *column* or *strut*.

The action of the internal stresses in a column is somewhat similar to a beam, with this difference—that in addition to the bending stress there is also present a direct compressive stress. As stated before, if the specimen is very short, the unit stress under a load of P pounds equals $S=\frac{P}{A}$. This unit compressive stress continues to exist as the length is increased, but there is an additional stress set up due to bending, so that the safe load that can be carried by a column gradually decreases for a given cross-section as the length increases.

A column will bend in a plane parallel to the shorter side of the column. Hence to make a column equally strong in either direction it is essential that the cross-section be sym-

metrical with respect to both axes. This is not always possible, owing to the difficulties of construction.

Columns are usually made of wood, cast iron, steel, or concrete. The load carried by a column will depend upon how the ends of the column are arranged. Three classes of columns are recognized.

I. Columns having *ends fixed,* as shown in Fig. 75.

II. Columns having one *end fixed* and the other *end free* as in Fig. 76.

III. Columns having both *ends free* or *pin ended* as shown in Fig. 77.

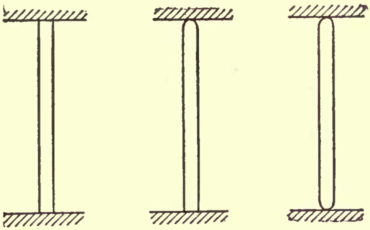


FIG. 75. FIG. 76. FIG. 77.

Columns in Class I are used in building and bridge construction. The column in such cases is rigidly secured to beams or concrete pedestals or similar fixed fastenings.

Columns in Class II are met with in the construction of machines, as, for example, the piston rod of an engine.

Columns in Class III are met with in bridge and machine construction. The connecting rod of a steam engine is a good illustration of a *pin-ended* column.

Experiment has shown that columns of Class I are the strongest, Class II is second, and Class III the weakest.

If the column is loaded eccentrically the safe load is materially effected, as a greater bending action is thereby induced.

PROB. 132. Calculate the theoretical angle of shear for cast-iron and wood specimens when subjected to a direct compressive

stress. The angle of repose of these materials can be found in any reference book on Mechanics.

Art. 42. Radius of Gyration

The moment of inertia of a cross-section is given by the equation $I = Ar^2$, where A is the area of the cross-section and r^2 is a term representing the average of the squares of the distances of each element of area from a given axis. For example, consider the case of an I beam; it is apparent that the moment of inertia can be figured about either the vertical axis—that is, an axis parallel to the web of the beam, or about a horizontal axis—that is, an axis parallel to the flanges of the beam. In structural work it is usual to refer to the moment of inertia about the vertical axis as I_{2-2}, and about the horizontal axis as I_{1-1}. These two values are always different, and in the case of a column the lesser value is to be used. When this lesser value is used the quantity r is called the *radius of gyration*, or

$$r = \sqrt{\frac{I}{A}}, \quad \ldots \ldots \quad (71)$$

where I is the least moment of inertia and A the area of the cross-section in inches. For example, the moment of inertia of a light 12-in. I beam about the axis 1–1 is 215.8, while about the axis 2–2 the moment of inertia is 9.5, and the area of the section is 9.26 sq. ins. Hence

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{9.50}{9.26}} = 1.01.$$

For a circle

$$r = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}. \quad \ldots \ldots \quad (72)$$

For a hollow circle of outside diameter d and inside diameter d_1

$$r=\sqrt{\frac{\frac{\pi}{64}(d^4-d_1^4)}{\frac{\pi}{4}(d^2-d_1^2)}}=\sqrt{\frac{d^2+d_1^2}{16}}, \quad . \quad . \quad (73)$$

For a square of side d

$$r=\sqrt{\frac{I}{A}}=\sqrt{\frac{\frac{d^4}{12}}{d^2}}=\sqrt{\frac{d^2}{12}}=\frac{d}{2\sqrt{3}} \quad . \quad . \quad . \quad . \quad (74)$$

For a hollow square

$$r=\sqrt{\frac{d^2-d_1^2}{12}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

The values of r for other sections can readily be determined provided the moment of inertia is known. No attempt should be made to determine the value of r independent of the moment of inertia.

In the design of columns the least moment of inertia is to be used, as the column will bend in a plane perpendicular to the axis about which the moment of inertia is the least.

PROB. 133. Find the radius of gyration for a hollow circular section whose outside diameter is 12 ins. and inside diameter 9 ins.

PROB. 134. Find the least radius of gyration for a light 12-in. channel beam. (Refer to Cambria or Carnegie handbook.)

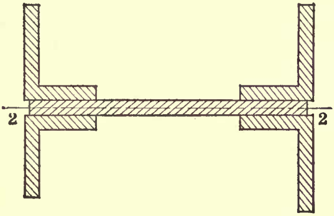


FIG. 78.

PROB. 135. Fig. 78 shows the cross-section of a plate and angle column. The angles are $4\times3\times\frac{5}{16}$. The plate is $14\times\frac{5}{16}$ in.

Calculate the radius of gyration about the axis 2–2. Refer to structural handbook for moment of inertia of the angles.

Art. 43. Rankine Column Formula

It is difficult to derive a rational formula for columns such as was done in the case of beams. The result has been that there are many empirical formulæ now in use for the design of columns. The formula developed by Rankine is one commonly used by engineers.

Fig. 79 shows a column under the action of a vertical load P, which tends to deflect the neutral axis of the column from the vertical line y–y to the curved line y–m–y. The vertical load P produces a uniform compressive stress of $\frac{P}{A}$ pounds on the cross-section of the column. Let this stress be represented by AB, Fig. 80, and be designated Sc, and let the line AC represent the least dimension of the column. It is evident that as the

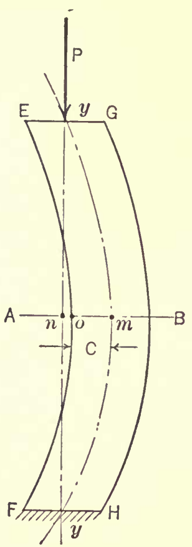


Fig. 79.

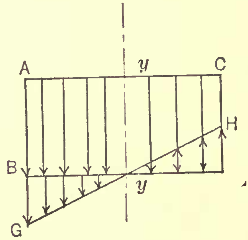


Fig. 80.

column deflects, the fibers on the side EF are put in compression, due to the bending action, and the fibers on the side GH are put in tension. Let this bending stress be designated S_b. The greater the deflection of the

column the greater will be the stress S_b. Let d = the lateral deflection of the column $= mn$. The bending moment on the section AB due to the vertical load P equals $P \times mn$, or $P \times d$, and the unit stress on the outer fibers of the column due to this bending moment equals $S_b = \frac{Mc}{I} = \frac{Pdc}{I}$. This bending stress varies from a maximum at the outermost fiber of the column to zero at the neutral axis. Let the line GH, Fig. 80, represent the variation in this fiber stress. Then it is apparent that the total stress on the outermost fiber at the point o is equal to the direct unit compressive stress due to bending. Let S = the maximum fiber stress at the point o, then,

$$S = S_c + S_b;$$

but

$$S_c = \frac{P}{A}, \quad \text{and} \quad S_b = \frac{Pdc}{I},$$

or

$$S = \frac{P}{A} + \frac{Pdc}{I}. \qquad (76)$$

It can be proven that the deflection d is proportional to the square of the length of the column, or $d \propto L^2$, hence $d = KL^2$ where K is a constant. From article 42, $I = Ar^2$, now substituting these values in Equation (76) gives

$$S = \frac{P}{A} + \frac{Pdc}{I};$$

$$= \frac{P}{A} + \frac{PKcL^2}{Ar^2}.$$

Let $Kc = q$, then,

$$S = \frac{P}{A} + \frac{P}{A} \cdot \frac{qL^2}{r^2}, \qquad (77)$$

or

$$S = \frac{P}{A}\left(1 + q\frac{L^2}{r^2}\right), \qquad (78)$$

where S equals the safe unit compressive stress, P equals the total load in pounds, A equals the area of the cross-section in square inches, q equals a constant (see Table XVI), L equals the length of the column in inches, and r equals the radius of gyration. If the dimensions of the column are known the safe load is found from the equation

$$P=\frac{AS}{1+q\frac{L^2}{r^2}}. \qquad (79)$$

For example, consider the case of a wooden column 8×8 ins. in cross-section which is 12 ft. long and has square ends. Let the safe unit compressive stress equal 1200. The area A in this case equals $8\times 8=64$ sq. ins. From Table XVI for a wooden column having square or fixed ends $q=\frac{1}{3000}$; $L=12$ ft. $=144$ ins. For a square section $r^2=\frac{d^2}{12}=\frac{8\times 8}{12}=5.33$, therefore

$$P=\frac{AS}{1+q\frac{L^2}{r^2}}=\frac{64\times 1200}{1+\frac{1}{3000}\cdot\frac{144^2}{5.33}}=\frac{76800}{2.3}=33{,}400 \text{ lbs.}$$

If a block of wood of the same cross-section were under a direct compressive stress the safe load would be $64\times 1200=76{,}800$ lbs. in place of 33,400 lbs. as above, which clearly shows the rapid decrease in load as the length of the column increases.

TABLE XVI

VALUES OF CONSTANT q

Material.	Both Ends Fixed.	One Fixed and One Pin End.	Both Ends Round.
Timber	1/3000	2/3000	4/3000
Cast iron	1/5000	2/5000	4/5000
Wrought iron	1/35000	2/35000	4/35000
Steel	1/25000	2/25000	4/25000

Prob. 136. Find the safe load that can be placed on a hollow yellow-pine column 12×12 outside dimensions and 8×8 inside dimensions; length of column 16 ft. Assume that the ends are fixed.

Prob. 137. Derive an expression for the diameter of a round column in terms of the load P, the area A, the length L, and the radius of gyration r. Use Equation (79) as a basis.

Art. 44. Cast-iron Columns

Cast-iron columns are frequently used in building construction owing to their cheapness, strength and ease of casting. The bases are cast integral with the column, so that they can readily be set in place. The cast-iron column is supported on a stone or concrete pier into which have been set holding-down bolts to prevent the column from moving in a lateral direction.

The Cambria Steel Co recommend the use of the following formula for determining the safe load in pounds per square inch that can be carried by a cast-iron column:

$$P=\frac{10000}{1+\frac{L^2}{800d^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

where P equals the safe load in pounds per square inch of area, L equals the length of the column in inches, and d equals the outside diameter in inches. For example, consider the case of a cast-iron column 6 in. in diameter and 12 ft. long; here $L=144$ and $L^2=20{,}736$; $d=6$ and $d^2=36$.

Hence

$$P=\frac{10000}{1+\frac{20736}{28800}}=5800 \text{ lbs. per sq in.}$$

The area of the column equals $.7854\times6^2=28.27$ sq. ins.

Hence the total load $=5800\times28.27=164{,}000$ lbs.

For the same data Rankine's formula gives

$$P=\frac{AS}{1+q\frac{L^2}{r^2}}=\frac{28.27\times 10000}{1+\frac{1}{5000}\cdot\frac{20736}{2.25}}=100\,000 \text{ (approx.)}.$$

These two values show the wide range in values given by various formulæ. For a solid cast-iron column d inches in diameter Equation (79) becomes

$$P=\frac{AS}{1+q\frac{L^2}{r^2}}=\frac{\frac{\pi d^2 S}{4}}{1+q\frac{L^2\times 16}{d^2}}. \quad . \quad . \quad . \quad . \quad (81)$$

This equation can be reduced to a biquadratic, but the author recommends that the known numerical values be substituted and the equation reduced to its simplest form. It will be found that the resulting equation will always be of the form

$$d^4-Ad^2+C=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

which can readily be solved by completing the square. For a hollow cast-iron column the problem is a little more complex. For example, let it be required to find the thickness of a hollow cast-iron column 20 ft., in length to carry a load of 164,000 lbs., the outside diameter being 10 ins. Assume the ends fixed and use a safe fiber stress of 10,000 lbs. per square inch. Let d equal the unknown inside diameter; then the area of the column equals

$$A=\frac{\pi}{4}(100-d^2) \text{ and } r^2=\frac{100+d^2}{16}.$$

Substituting these values in Equation (79) gives

$$P=\frac{AS}{1+q\frac{L^2}{r^2}}=\frac{\frac{\pi}{4}(100-d^2)\times 10000}{1+\frac{1}{5000}\cdot\frac{57600\times 16}{(100+d^2)}}=\frac{\frac{\pi}{4}(100-d^2)\times 10000}{1+\frac{184.3}{(100+d^2)}}.$$

Now reduce the right-hand member to the form

$$\frac{\frac{\pi}{4}(100-d^2)(100+d^2)\times 10000}{284.3+d^2}=\frac{7854\times(10000-d^4)}{284.3+d^2},$$

but

$$P=164000,$$

hence

$$164000=\frac{7854(10000-d^4)}{284.3+d^2},$$

$$d^4+20.9d^2-4059=0;$$

$$d^2=54.2;$$

$$d=7.4 \text{ in.}$$

Hence the thickness of the metal $=\frac{10-7.4}{2}=1.3$ ins.

Rankine's formula is gradually being replaced by the straight-line formula, so called because when values are substituted in the formula and the results plotted to scale the resulting curve is a straight line. For square-end timber columns of long-leaf yellow pine the New York building laws specify the Equation,

$$\frac{P}{A}=1000-18\frac{L}{D}, \quad \ldots\ldots \quad (83)$$

where D is the least transverse dimension.

For cast-iron columns the straight-line formula is given as,

$$\frac{P}{A}=10000-40\frac{L}{r}, \quad \ldots\ldots \quad (84)$$

where $\frac{L}{r}$ is the ratio of the length to the least radius of gyration.

For steel columns New York City requires the following:

$$\frac{P}{A}=15200-58\frac{L}{r}. \qquad \qquad (85)$$

PROB. 138. Find the safe load by New York law on a 6×8 yellow-pine post 12 ft. long.

PROB. 139. Figure the safe load for Prob. 138 by using Rankine's formula.

PROB. 140. A cast-iron column 18 ft. long is to carry 200,000 lbs. Assuming the ends fixed, find the outside diameter if the inside diameter is 8 ins., so that the maximum fiber stress shall not exceed 10,000 lbs. per sq. in.

ART. 45. STRUCTURAL COLUMNS

For structural work columns are made of steel I beams or combinations of the other rolled shapes. When an I beam is used as a column the safe load is determined by using any of the standard column formulæ, but in every case the least radius of gyration must be used. Fig. 81 shows the arrangement of plates, angles, channels, and Z bars in various types of columns. Fig 81*a* and 81*b* show two forms of plate and angle column. To determine the safe load, the moment of inertia must be taken about the 1–1 axis. In figuring the moment of inertia it is usual to neglect the rivet holes. Fig. 81*c* shows a form of latticed channel column. In figuring the safe load the moment of inertia of the channels is used, and it is assumed that the lattice work merely ties the columns together and prevents lateral deflection. Figs. 81*d* and 81*e* show forms of plate and channel column. Fig. 81*f* shows a form of Z-bar column.

In designing a structural column it is desirable to so space the various elements that the moments of inertia about the axes 1–1 and 2–2 will be as near equal as possible.

Cambria Steel Co. suggest using Gordon's formula for determining the safe load on steel columns, using a factor of safety of four, so that the unit stress in Equation (86), Art. 46, will equal 12,500.

For example, a steel column of the form shown in Fig. 81*d* is made up of two 12-in. Cambria channels weighing 25 lbs. per foot and two steel plates $14 \times \frac{1}{2}$ in. By reference to Cambria handbook it is found that the moment of inertia is the least about the axis 2–2 and equals 524.3. In case the value of the moment of inertia is unknown it may be figured as follows: Let Fig. 82 represent the cross-section of the

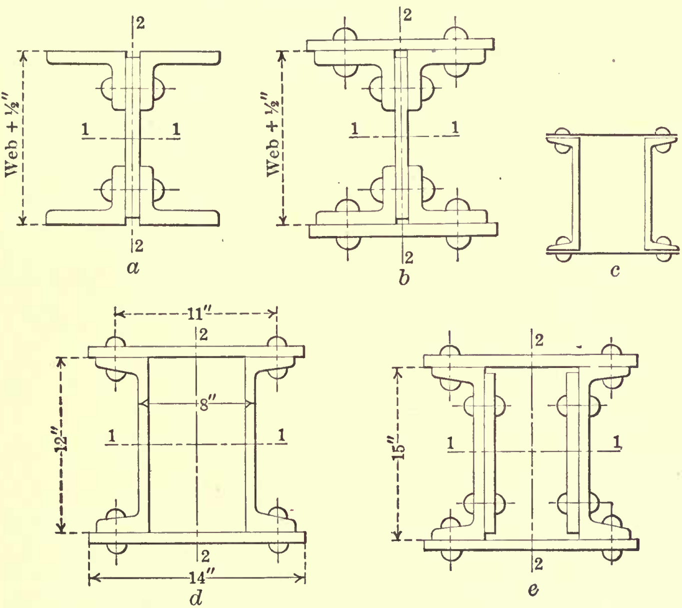


Fig. 81.

column and assume the flanges to be of uniform thickness equal to the average thickness of the top and bottom of the flanges. This average thickness equals 0.5 in. Now divide the section above the axis 2–2 into the rectangles I, II, III, IV, and V, having dimensions as shown in Fig. 82. The computations for the moment of inertia are given in Table XVII. The moment of inertia of the entire figure equals

twice this amount, as the section is symmetrical with respect to the axis 2–2. It will be noted that the value of I as computed equals 563.78, while that given in Cambria is 524.3. This difference is due to the fact that the channels are not set flush with the edge of the plate. In making calculations for I the exact dimensions of the column are to be used.

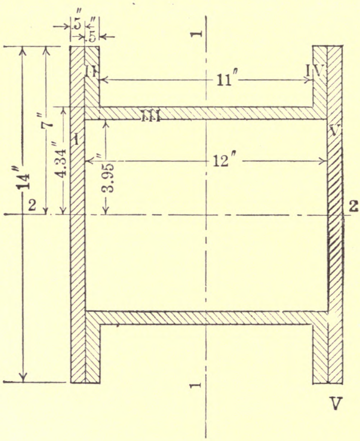


Fig. 82.

TABLE XVII

Mark.	b	h	h_1	h^3	h_1^3	$h^3 - h_1^3$	$b(h^3 - h_1^3)$
I	.5	7	0	343	0	343	171.50
II	.5	7	3.95	343	61.63	281.37	140.68
III	11	4.34	3.95	81.75	61.63	20.12	221.32
IV	.5	7	3.95	343	61.63	281.37	140.68
V	.5	7	0	343	0	343	171.50
						3	845.68
					Above	axis	281.89
					Below	axis	281.89
						$I_{2-2}=$	563.78

After the least value of I has been determined the radius of gyration is found from the formula $r=\sqrt{\frac{I}{A}}$. In this case $r=\sqrt{\frac{563.78}{28.7}}=4.4$. Based on the value given in Cambria $r=4.27$.

Using Gordon's formula and assuming length of column equals 20 ft., the total load that can be carried by the column equals

$$P=\frac{A\times 12500}{1+\frac{(12L)^2}{36000r^2}} \quad \text{or} \quad P=\frac{28.7\times 12500}{1+\frac{240^2}{36000\times 18.23}}=330{,}000 \text{ lbs.}$$

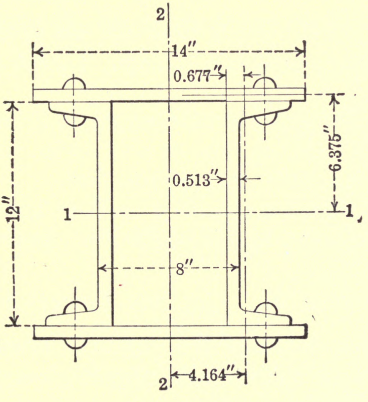


Fig. 83.

Another method of figuring the moment of inertia of a compound section is as follows: Let Fig. 83 represent the cross-section of a plate and channel column. From the handbook the moment of inertia of the two channels about the axis 1-1 equals $2\times 161.7=323.4$. The channels weigh 30 lbs. per ft., and the plates are $14\times\frac{3}{4}$ in. The moment of inertia of the plate relative to its own gravity axis equals

$\frac{14\times\frac{3}{4}^3}{12}=.98.$ This moment must be transferred to the axis 1–1 by means of the reduction formula explained in Art. 23, or

$$I_{1\text{-}1}\text{ (of plate)}=I_g+Ad^2=.98+10.5\times.375^2=427.7.$$

Hence the total moment of inertia of the section relative to the axis 1–1 equals.

$I_{1\text{-}1}$ for two 12-in. channels $=2\times161.7 \qquad =323.4$

$I_{1\text{-}1}$ for two 14-in.$\times\frac{3}{4}$-in. plates $=2\times427.7 \qquad =\frac{855.4}{1178.8}$;

The ratius of gyration equals $\sqrt{\frac{I}{A}}=\sqrt{\frac{1178.8}{38.64}}=5.52.$

Prob. 141. Find the safe load that can be carried by a light 12-in. I beam when used as a column. Length of column equals 25 ft.

Prob. 142. A column of the form shown in Fig. 81a is made up of four angles $6\times4\times\frac{1}{2}$ in. and of a $12\times\frac{1}{2}$ in. web plate. If the length of column is 22 ft., find the safe load.

Prob. 143. Select a suitable I beam to be used as a column 24 ft. long to support a load of 75,000 lbs. Assume ends fixed.

Art. 46. Column Formulæ

In designing columns the engineer must so proportion the metal that the column will support a maximum load with a minimum of metal. This means selecting the elements of the column so that the moment of inertia of the section shall be a maximum for a given weight. Many formulæ have been suggested to simplify the calculations for columns. The Cambria Steel Co. recommend Gordon's formula for steel columns with fixed ends, which is,

$$P=\frac{50000}{1+\frac{(12L)^2}{36000r^2}}, \qquad \ldots\ldots \quad (86)$$

where P equals the maximum load in pounds per square inch which the column can support. This value is to be divided by the desired factor of safety, which is taken as four; L equals the length of the column in feet, and r equals the least radius of gyration of the section. The total safe load which the column can support equals the load P times the cross-sectional area of the column, divided by the desired factor of safety.

Many of the formulæ take the form of the equation of a straight line. These are in the form $P=\left(A-C\frac{L}{r}\right)$, where A and C are constants, L equals the length in inches and r equals the least radius of gyration. For steel columns with fixed ends the building laws of New York, Philadelphia, and Boston require the use of the following formulæ:

New York

$$P=15200-58\frac{L}{r}. \qquad (87)$$

Philadelphia

$$P=\frac{16250}{1+\frac{L^2}{11000r^2}}. \qquad (88)$$

Boston

$$P=\frac{16000}{1+\frac{L^2}{20000r^2}}. \qquad (89)$$

The American Railway Engineers Association recommend

$$P=16000-70\frac{L}{r} \qquad (90)$$

In all cases P equals the safe unit stress in pounds per square inch of cross-sectional area of the column. It is customary to limit the maximum ratio of $\frac{L}{r}$ to 100 to 120.

Another formula which is sometimes used is that suggested by Euler, which is

$$P=\frac{4\pi^2 EI}{L^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

where P equals the load in pounds, E equals the modulus of elasticity of the material, I equals the least moment of inertia of the section, and L equals the length.

Prob. 144. Compare the safe load that can be carried by a 12-in. steel I beam when used as a column 20 ft. long, using Equations (87), (88), and (89).

Prob. 145. Find the safe load that can be carried by a column of the form and dimensions shown in Fig. 81*d*. Length of column is 24 ft. Channels weigh 30 lbs. per lin ft. Plates are 1 in. thick. Figure the load by using both Equations (79) and (90). Compare the results.

Art. 47. Review Problems

Prob. 146. By use of Equation (80) find the safe load that can be placed on a hollow cast-iron column 22 ft. in length. Outside diameter of column is 14 ins. and the inside diameter 11 ins.

Prob. 147. A hollow cast-iron column supports a load of 330,000 lbs. The column is 20 ft. in length. Find the thickness of the metal if the outside diameter is 12 ins. Use Rankine's formula.

Prob. 148. A steel column is made up of two 15-in. columns weighing 40 lbs. per ft., and two steel plates 20 ins. wide and $\frac{1}{2}$ in. thick. The channels are placed back to back. Find the safe load if the column is 24 ft. long. Use Equation (87).

Prob. 149. The diameter of the cylinder of an engine is 24 ins. The maximum unbalanced steam pressure is 120 lbs. per sq. in. Find the diameter of a steel piston rod, assuming the length of the rod to be 6 ft.

Prob. 150. Determine the diameter of the connecting rod of the engine in Prob. 149, assuming the stroke of the engine to be

42 ins. and the length of the connecting rod to be 9 ft. (NOTE.—determine the maximum thrust on the connecting rod.)

PROB. 151. A 9×12 in. piece of yellow pine 16 ft. long is used as a column with fixed ends. Use Equation (83).

PROB. 152. Find the safe load that can be carried by a column of the cross-section shown in Fig. 83. Length of column is 36 ft.

PROB. 153. Find the radius of gyration of a column whose section is made up of four $5 \times 3\frac{1}{2} \times \frac{1}{2}$ angles and one plate $10 \times \frac{1}{2}$ arranged as shown in Fig. 81*a*.

PROB. 154. Find the moment of inertia of the column section shown in Fig. 81*b*. The angles are $6 \times 4 \times \frac{1}{2}$. The flange plates are $14 \times \frac{3}{8}$, and the web plate is $14 \times \frac{3}{8}$ in.

PROB. 155. In Prob. 154 determine the least radius of gyration of the section.

CHAPTER IX

RIVETED JOINTS

Art. 48. Cast-iron Pipe

When a cylindrical vessel is subjected to an internal pressure of P pounds per square inch, the required thickness of metal is found from the formula,

$$PD = 2St, \qquad (92)$$

or

$$t = \frac{PD}{2S}. \qquad (93)$$

where P equals the internal pressure in pounds per square inch, D equals the inside diameter in inches, t equals the thickness of the metal in inches and S equals the safe working stress in pounds per square inch.

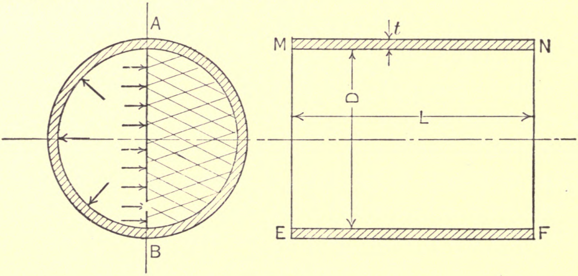


Fig. 84.

In a cylindrical or spherical vessel the internal pressure acts in a radial direction, or normal to the surface of the vessel. In Fig. 84 consider one-half of the pipe filled with a solid substance. It is evident the pipe will tend to break

along the lines MN and EF. The total pressure tending to burst the pipe equals the unit pressure P, times the projected area LD of the pipe, or the total pressure equals PLD. The strength of the metal resisting this pressure equals $2tLS$, assuming that the pipe is equally strong on either side. For equilibrium to exist, the internal pressure and the strength of the metal must be equal, or,

$$PDL = 2tLS,$$

hence,

$$PD = 2St.$$

The above formula is applicable only when the inside diameter of the pipe is at least ten times the thickness of the metal.

Large water and soil pipes are made of cast iron. Owing to the imperfections due to casting and to the structure of the cast iron a large factor of safety is to be used in determining the thickness of metal. This is usually taken from 12 to 16.

For example, consider a pipe 24 ins. in diameter subjected to an internal pressure of 100 lbs. per sq. in. The tensile strength of cast iron is 20,000, so that using a factor of safety of 12 gives a unit working stress of approximately 1600 lbs. per sq. in. Substituting this value in Equation (93) gives

$$t = \frac{PD}{2S} = \frac{100 \times 24}{2 \times 1600} = \tfrac{3}{4} \text{ in.}$$

The transverse section of a pipe is twice as strong as the longitudinal section. The force tending to burst the pipe on a transverse section equals $\frac{\pi D^2}{4} \times P$. The strength of the

metal resisting this force equals the area of the cross-section of the metal in the pipe, or πDtS, hence

$$\frac{\pi D^2}{4}P=\pi DtS,$$

or

$$PD=4St \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

Equation (94) can be used to determine the thickness of a sphere.

PROB. 156. A cast-iron water pipe 36 ins. in diameter is subjected to a head of 250 ft. Find the thickness of the metal, using a factor of safety of 12.

PROB. 157. Find the safe working pressure that can be placed on a 48-in. cast-iron water pipe if the thickness of the metal is 2 ins.

ART. 49. LAP JOINTS

The simplest method of joining two plates is by means of a riveted joint. The rivets are made of iron or steel, and consist of an upset end called the *head*, and a long cylindrical part called the *shank*. Rivets in general are placed at right angles to the forces tending to cause them to fail.

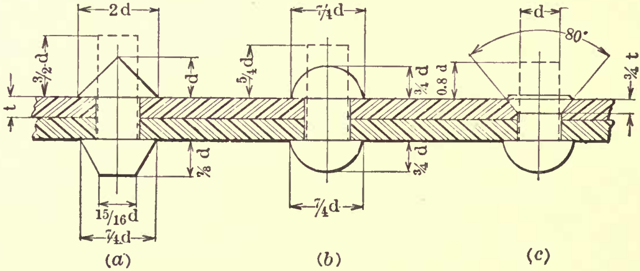


FIG. 85.

Fig. 85 shows various forms of rivet points. Fig. 85*a* is called the cone or steeple point; Fig. 85*b* is known as the *button* head, while Fig. 85*c* shows a form of countersink rivet.

In punching the plate the rivet hole is made $\frac{1}{16}$ in. larger than the diameter of the rivet.

In designing a riveted joint for structural work only the question of strength need be considered, but in the case of boilers, tanks, etc., the joint must be both strong and water-tight.

The simplest form of riveted joint is that shown in Fig. 86, where one plate overlaps the other, and the plates are secured in place by a single row of rivets. Such a joint is referred to as a *single-riveted* lap joint. This joint may fail

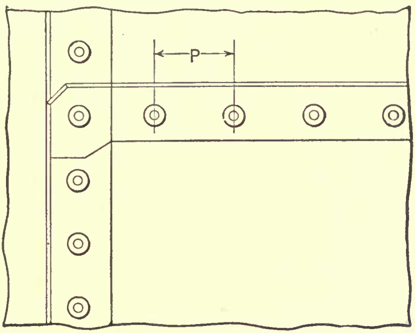


FIG. 86.

in one of several ways. First, the rivets may shear; second, the plate may tear between the rivet holes; third, the rivets may shear the plate; fourth, the rivets may crush the plate in front of the rivet.

The *pitch* of a riveted joint is the distance from center to center of rivets in the outer course (provided there are several rows of rivets).

The distance from the edge of the rivet hole to the edge of the plate is called the *margin*.

The distance between the edges of two plates which overlap is termed the *overlap*.

In no case can the strength of a riveted joint be greater

than the strength of the solid plate. It is usual to investigate a riveted joint for failure of the rivets in shear and for failure of the punched plate in tension. The efficiency of a riveted joint is based on the strength of the solid plate. The *efficiency of the plate* is the ratio of the strength of the punched plate to the strength of the solid plate.

The *efficiency* of the *rivets* is the ratio of the strength of the rivets to the strength of the solid plate. In design both of these efficiencies are calculated and the lesser value used.

For example, in the case of a single-riveted lap joint, as shown in Fig. 86, let P equal the pitch in inches, d equal the diameter of the rivet after being driven or the diameter of the rivet hole, T equal the tensile strength of the plate in pounds per square inch, S equal the shearing strength of the rivets in pounds per square inch, and t equal the thickness of the plate in inches. Then the strength of the solid plate, for a distance equal to the pitch, is $T \times P \times t$, and the strength of the punched plate is $T \times (P-d) \times t$, hence the efficiency of the plate equals

$$\frac{T \times (P-d)}{T \times P \times t} = \frac{P-d}{P}, \qquad (95)$$

and the efficiency of the rivets equals

$$\frac{\dfrac{\pi d^2 \times S}{4}}{T \times P \times t}. \qquad (96)$$

The shearing strength of the rivets is about three-quarters the tensile strength of the plate, or $S = \frac{3}{4}T$. Substituting this value in Equation (96) gives the efficiency of the rivets equal

$$\frac{\dfrac{\pi d^2}{4} \times \dfrac{3}{4}T}{T \times P \times t} = \frac{.59d^2}{P \times t}.$$

If the plate is to be equally strong with the rivets, the efficiency of plate and rivets must be equal, or,

$$\frac{.59d^2}{P \times t} = \frac{P-d}{P},$$

or

$$(P-d) = \frac{.59d^2}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (97)$$

From Equation (97) the required pitch may be found for a given thickness of plate and diameter of rivet.

For boiler work the American Society of Mechanical Engineers recommend the following method of determining the efficiency of a single-riveted lap joint; let A equal the strength of the solid plate equal $P \times T \times t$; let B equal the strength of the plate between rivet holes equal $(P-d) \times T \times t$; let C equal the shearing strength of one rivet in single shear equal $N \times S \times a$, where N equals the number of rivets in single shear in a unit of length, $S =$ the shearing strength of a rivet in single shear and a equals the area of the rivet after driving; and let D equal the crushing strength of the plate in front of the rivet equals $d \times t \times c$, where c equals the crushing strength of the plate. To find the efficiency of the joint, calculate the values of B, C, and D; divide the lesser value by the value of A, and the quotient will be the required efficiency.

For example, let the pitch $P = 1\frac{5}{8}$ in., $t = \frac{1}{4}$ ins., diameter of rivet hole equal $\frac{11}{16}$, $T = 55{,}000$, $S = 44{,}000$, and $C = 95{,}000$.

Then $A = P \times T \times t = 1\frac{5}{8} \times 55{,}000 \times \frac{1}{4} = 22{,}343.$

$$B = (P-d) \times T \times t = (1.625 - .6874) \times 55{,}000 \times \tfrac{1}{4} = 12{,}890;$$

$$C = N \times S \times a = 1 \times 44{,}000 \times .3712 = 16{,}332;$$

$$D = d \times t \times c = .6875 \times \tfrac{1}{4} \times 95{,}000 = 16{,}328.$$

The value of B is less than that of C or D, hence the efficiency of the joint $= \frac{B}{A} = \frac{12890}{22343} = 57.6$ per cent.

For complete analysis of this method the student is referred to Vol. XXXVI of the Transactions of the A.S.M.E.

In the case of a double-riveted lap joint, as shown in Fig. 87, the same method can be applied. In figuring the value of C in this case N is equal to two, as there will be two

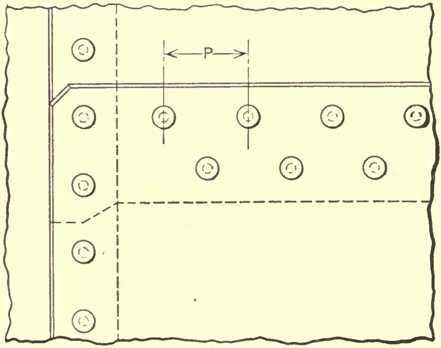


Fig. 87.

rivets in single shear, and likewise $D = 2 \times d \times t \times c$, as there are two rivets tending to crush the plate.

Prob. 158. Find the efficiency of a double-riveted lap joint where the pitch of the rivets is 3 ins., the diameter of the rivets $\frac{11}{16}$ in. Assume $S = 44{,}000$, $C = 95{,}000$, and $T = 55{,}000$.

Art. 50. Butt Joints

Lap joints are used in making the girth seams of boilers and tanks. The longitudinal seams are formed by butting the two ends of the plate together and securing them in place by use of an inside and outside cover plate, as shown in Fig. 88. This is referred to as a butt joint. In Fig. 88 the joint is double riveted, in Fig. 89 the joint is triple riveted, and in Fig. 90 the joint is quadruple riveted. The more rows of rivets the more nearly will the efficiency of the joint equal that of the solid plate. Theoretically a rivet should be twice

as strong in double shear as in single shear, but practically such is not the case. The A.S.M.E. recommend the following values for iron and steel rivets in single and double shear:

Iron rivets in single shear	=38,000;
Iron rivets in double shear	=76,000;
Steel rivets in single shear	=44,000;
Steel rivets in double shear	=88,000.

In calculating the efficiency of a butt joint the pitch is taken as the distance between centers of the rivets in the

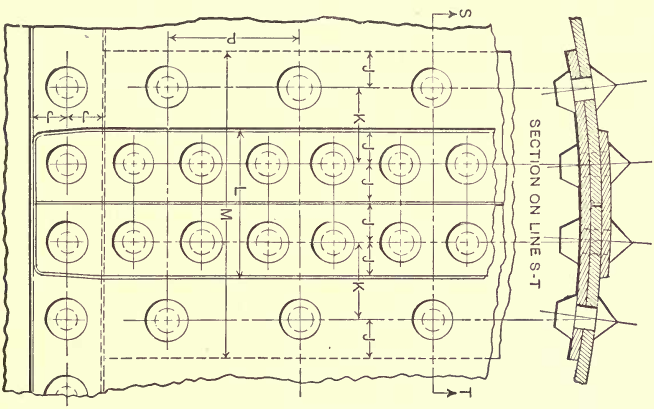


FIG. 88.

outer course. It will be noted that in the double-riveted butt joint the pitch of the inner row is one-half that of the outer row. In the triple-riveted butt joint the pitch of the second and third rows is one-half that of the outer row, or course.

The triple-riveted butt joint is commonly used in making the longitudinal seam of the steam drum of water-tube boilers. The method of determining the efficiency is as follows:

Let A equal the strength of the solid plate equal $P \times t \times T$;

B equal the strength of the plate between rivet holes in the outer course equal $(P-d) \times t \times T$;

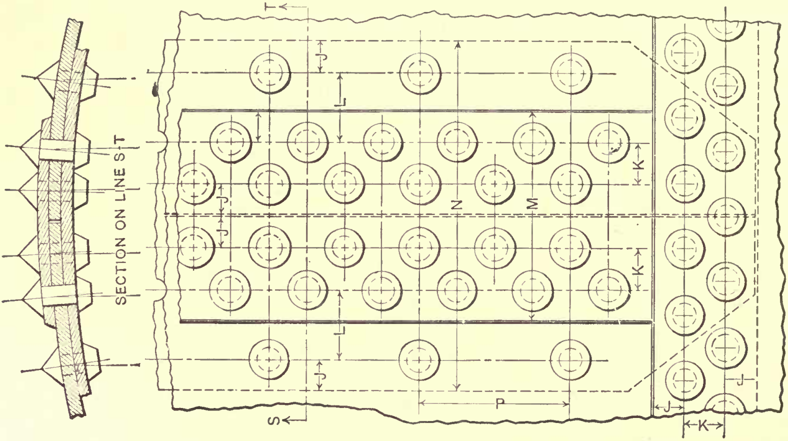


FIG. 89.

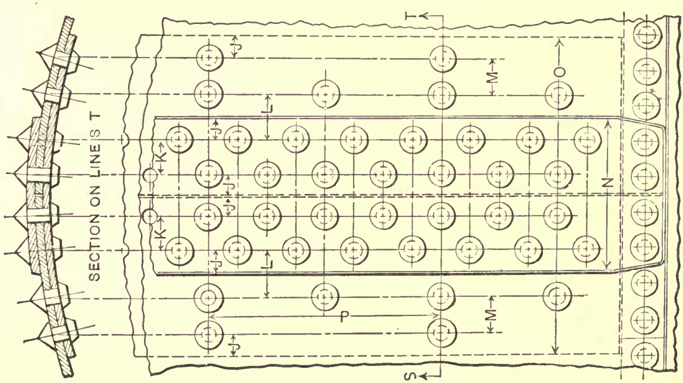


FIG. 90.

C equal the shearing strength of four rivets in double shear, plus the shearing strength of one rivet in single shear equal $N \times S \times a + n \times s \times a$;

D equal the strength of the plate between rivet holes in the second row, plus the shearing strength of one rivet in single shear in the outer row equal $(P-2d) \times t \times T + n \times s \times a$;

E equal the strength of the plate between rivet holes in the second row, plus the crushing strength of the butt strap in front of one rivet in the outer row equal $(P-2d) \times t \times T + d \times b \times c$, where

S equals the strength of rivet in double shear;

N equals the number of rivets in double shear in the unit of length and b equal the thickness of the butt strap.

The efficiency of the joint is determined by finding the values of B, C, D, and E and dividing the least of these values by A. For example, in a triple-riveted butt joint with inside and outside cover plates or butt straps, the thickness of plate is $\frac{3}{8}$ in., $P=6.5$ ins., $b=\frac{5}{16}$ ins., $d=\frac{13}{16}$ ins.

Here $A = P \times t \times T = 6.5 \times .375 \times 55{,}000 = 134{,}062$;

$B = (P-d) \times t \times T = (6.5-.8125) \times .375 \times 55{,}000 = 117{,}304$;

$C = N \times S \times A + n \times s \times a = 4 \times 88{,}000 \times .5185 + 1 \times 44{,}000 \times .5185 = 205{,}326$;

$D = (P-2d) \times t \times T + n \times s \times a = (6.5-2 \times .8125) \times .375 \times 55{,}000 + 1 \times 44{,}000 \times .5185 = 123{,}360$;

$E = (P-2d) \times t \times T + d \times b \times c$
$= (6.5-2 \times .8125) \times .375 \times 55{,}000 + .8125 \times .3125 \times 95{,}000 = 124{,}667$.

Here the value of B is the least, hence the efficiency equals

$$\frac{B}{A} = \frac{117304}{134062} = 87.5 \text{ per cent.}$$

In structural work butt joints are used to join together various sections of beams, girders, columns, etc. In such cases a sufficient number of rivets must be provided to take care of the vertical shear at any given point. The Cambria

Steel Co. suggest the following rules for rivet spacing in bridge and structural work:

"The pitch or distance from center to center of rivets should not be less than three diameters of the rivet. In bridge work the pitch should not exceed 6 ins., or sixteen times the thickness of the thinnest outside plate, except in special cases hereafter noted. In the flanges of beams and girders where plates more than 12 ins. wide are used, an extra line of rivets with a pitch not greater than 9 ins. should be driven along each edge to draw the plates together.

"At the ends of compression members the pitch should not exceed four diameters of the rivet for a length equal to twice the width or diameter of the member.

"In the flanges of girders and chords carrying floors, the pitch should not exceed 4 ins.

"For plates in compression the pitch in the direction of the line of stress should not exceed sixteen times the thickness of the plate, and the pitch in a direction at right angles to the line of stress should not exceed thirty-two times the thickness, except for cover plates of top chords and end posts, in which the pitch should not exceed forty times their thickness.

"The distance between the edge of any piece and the center of the rivet hole should not be less than $1\frac{1}{4}$ ins. for $\frac{3}{4}$ in. and $\frac{7}{8}$-in. rivets, except in bars less than $2\frac{1}{2}$ ins. wide; when practicable it should, for all sizes, be at least two diameters of the rivet and should not exceed eight times the thickness of the plate.

"Minimum spacing is generally used in pin plates, at ends of columns, girders, etc.

"In figuring clearance of rivets for special cases, allow $\frac{3}{8}$ in. in addition to diameter of head."

PROB. 159. In a quadruple-riveted joint $P = 15$ ins., $t = \frac{1}{2}$ in., $b = \frac{7}{16}$ in., $d = \frac{15}{16}$ in. Find the values of A, B, C, and D. Determine the efficiency of the joint.

Art. 51. Application to Boilers

The formula for determining the thickness of a pipe is not applicable to a vessel which is made up of rolled plates joined together by riveted joints. In such cases the efficiency of the joint bears an important part in fixing the thickness of metal. Various formulæ have been suggested from time to time, and the one now recommended by the A.S.M.E. is the following:

$$P=\frac{\text{T.S.}\times t\times E}{R\times \text{F.S.}}, \qquad \ldots \ldots \quad (98)$$

where T.S. equals ultimate tensile strength stamped on shell plates, pounds per square inch;
t equals minimum thickness of shell plates in the weakest course in inches;
E equals efficiency of the longitudinal joint or of ligaments between two holes (whichever is the least);
R equals the inside radius of the weakest course of the shell or drum, in inches;
F.S. equals the factor of safety, or the ratio of the ultimate strength of the material to the allowable stress;
P equals the allowable working steam pressure in lbs. per sq. in.

For new boilers the factor of safety is to be taken as five.

This formula is derived by considering that efficiency equals $\frac{\text{output}}{\text{input}}$. The output of the riveted joint is holding the pressure P, against a given radius R, or equals $2PR$. The input is the strength of the punched plate equals $2tS_t$ for a unit of length, hence

$$E=\frac{2PR}{2tS_t}=\frac{PR}{tS_t};$$

but

$$S_t=\frac{\text{T.S.}}{\text{F.S.}},$$

hence

$$E=\frac{P\times R\times \text{F.S.}}{t\times \text{T.S.}}.$$

or

$$P=\frac{\text{T.S.}\times t\times E}{R\times \text{F.S.}}.$$

The required thickness of plate is given by the formula

$$t=\frac{P\times R\times \text{F.S.}}{\text{T.S.}\times E}. \quad . \quad . \quad . \quad . \quad . \quad (99)$$

For example, consider a 42-in. steam drum such as is used on a B. & W. water-tube boiler. Assume boiler pressure equal 125 lbs. per sq. in. gauge, and assume the longitudinal seam to be a triple-riveted butt joint having an efficiency of 85 per cent. The tensile strength of the plate is 55,000 lbs. per sq. in. As stated above in new installations a factor of safety of five is to be used, hence the thickness in this case equals

$$t=\frac{125\times 21\times 5}{55000\times .85}=.28 \text{ in.}$$

From Equation (99) it is noted that the lower the efficiency of the joint the greater will be thickness of metal required. In large fire-tube boilers of the locomotive type it is desirable to use a joint having as high an efficiency as possible.

Example. A locomotive boiler has a maximum inside diameter of 68 ins. The maximum steam pressure is 220 lbs. per sq. in. The longitudinal seam is a triple-riveted butt joint with an efficiency of 87 per cent. Tensile strength of the shell plate is 56,000 lbs. per sq. in. The thickness of

the shell is $\frac{11}{16}$ in. Determine the factor of safety. Using Equation (99) and transposing the terms gives,

$$\text{F.S.} = \frac{\text{T.S.} \times t \times E}{P \times R};$$

$$= \frac{55000 \times 11 \times .87}{220 \times 16 \times 34} = 4.4.$$

In locomotive work a factor of safety of 4.5 is permissible.

PROB. 160. A Scotch marine boiler is 16 ft. in diameter, and carries a pressure of 175 lbs. per sq. in. Using a shell plate having a tensile strength of 56,000 lbs. per sq. in., find the thickness of the shell. Longitudinal seam is a triple-riveted butt joint with an efficiency of 88 per cent.

PROB. 161. An air tank carries a pressure of 80 lbs. per sq. in. The tank is 48 ins. in diameter. Longitudinal seam is a double-riveted butt joint having an efficiency of 72 per cent. Find the proper thickness of shell plate if tensile strength of plate is 50,000 lbs. per sq. in.

ART. 52. PRACTICAL PROBLEMS

PROB. 162. Find the thickness of a 16-in. cast-iron standpipe which is subjected to a head of water of 250 ft. Assume a steady stress.

PROB. 163. Find the pressure required to burst a cast-iron cylinder 12 ins. in diameter and 1.5 ins. thick.

PROB. 164. Given a double-riveted lap joint. Thickness of plate equals $\frac{1}{2}$ in. Diameter of rivets equals $\frac{7}{8}$ in. The strength of the punched plate to equal the strength of the rivets. Assume $S_s = \frac{3}{4}S_t$. Find the pitch of the rivets and the efficiency of the joint.

PROB. 165. Diameter of a steel boiler is 65 ins. The thickness of the shell is $\frac{11}{16}$ in. Steam pressure is 180 lbs. per sq. in. gauge. If the efficiency of the longitudinal seam is 87 per cent, find the factor of safety.

PROB. 166. In a single-riveted lap joint the pitch is $2\frac{1}{8}$ in., diameter of rivets is $\frac{3}{4}$ in. Diameter of shell is 36 in. If the steam

pressure is 100 lbs. per sq. in., find the unit shearing stress on the rivets, and the unit tensile stress on the plate. The lap joint forms the girth seam.

PROB. 167. A Scotch marine boiler is 16 ft. in diameter. Longitudinal seam is a triple-riveted butt joint having an efficiency of 85 per cent. Maximum steam pressure is 125 lbs. per sq. in. Find thickness of the shell.

PROB. 168. In a quadruple-riveted joint, the pitch of the rivets in the outer row is 18 ins., thickness of plate is $\frac{13}{16}$ in., thickness of butt straps is $\frac{9}{16}$ in., diameter of rivets is $1\frac{1}{8}$ in., pitch in the second row is 9 ins., and in the third and inner rows is $4\frac{1}{2}$ ins. Tensile strength of the plate is 55,000 lbs. per sq. in. Draw the riveted joint to scale and figure the efficiency of the joint. Width of inner butt strap is 26 ins. and of outer butt strap is $12\frac{1}{4}$ ins.

PROB. 169. In a triple-riveted butt joint the pitch in the outer row is 8 ins. Thickness of plate is $\frac{1}{2}$ in. and of butt straps is $\frac{7}{16}$ in. Diameter of rivets is $\frac{7}{8}$ in. Diameter of boiler is 42 ins. Find the safe working pressure in pounds per square inch. Tensile strength of plate is 56,000 lbs. per sq. in.

PROB. 170. A return tubular boiler is 84 ins. in diameter and carries a pressure of 125 lbs. per sq. in. in gauge. The longitudinal seam is a triple-riveted butt joint. Efficiency of joint is 84 per cent. Find the thickness of the metal.

PROB. 171. (*a*) In the above problem design a double-riveted lap joint for the girth seam. (*b*) Which is the weaker, the longitudinal or the girth seam?

CHAPTER X

COMBINED STRESSES—RESILIENCE

ART. 53.—CRANE HOOKS—FLEXURE AND TENSION

THERE are many cases of problems involving a combination of stresses. The mathematics involved in some of these is too complicated for this text, so the student is referred to higher works on Applied Mechanics. There are, however, several simple cases. One of these is the combination of flexure or bending and pure tension such as found in the design of crane hooks.

Fig. 91 shows a common form of hook. The section CD is under pure tension, and the unit stress on this section is found from the formula $S=\frac{W}{A}$, where W is the load in pounds and A the cross-section in square inches at the section CD.

The area EF is subjected to pure shear, hence the unit shearing stress is $\frac{W}{A_1}$, where A_1 is the area at section EF.

The section AB is subjected to a pure tensile stress due to the load W and in addition is subjected to a bending action due to the fact that the neutral axis of the section AB is not in line with load W. This bending action produces a tension in the part to the right of the neutral axis and a compression in the part to the left of the section. Hence the maximum stress is in the outermost fiber at the point B.

Let the tensile stress due to the pure tension equal

$S_t=\frac{W}{A}$, and let the tensile stress due to bending equal $S_b=\frac{Mc}{I}$, then the total tensile stress on the outer fiber of the section AB is $S=S_t+S_b=\frac{W}{A}+\frac{Mc}{I}$. The bending

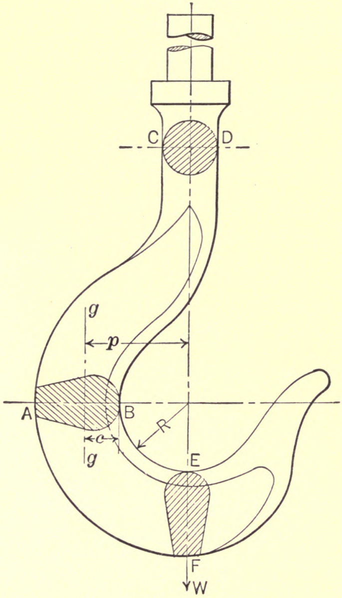


FIG. 91.

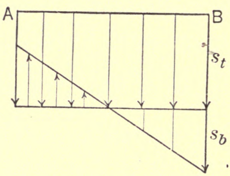


FIG. 91*a*.

moment $M=Wp$, where p is the distance from the line of action of the weight W to the neutral axis of the section AB. Hence the formula for the design of the section AB becomes,

$$S=\frac{W}{A}+\frac{Wpc}{I}. \qquad \qquad (100)$$

where S equals safe unit working stress in tension, W equals the load in pounds, c equals the distance from the neutral axis

to the outermost fiber at B, and I equals the moment of inertia of the section relative to the axis g–g.

EXAMPLE. Assume the hook to be $1\frac{1}{2}$ ins. in diameter at the section AB; let the distance $p=2$ ins. The hook is made of wrought iron; let $S=8000$ lbs. per sq. in. Find the safe load that can be carried by the hook.

To find this load substitute the above values in Equation (100) thus:

$$S=\frac{W}{A}+\frac{Wpc}{I},$$

where

$$A=.7854d^2=1.77;\ p=2 \text{ ins.},$$

$$c=\frac{d}{2}=.75;\ I=\frac{\pi d^4}{64}=0.25,$$

hence,

$$8000=\frac{W}{1.77}+\frac{W\times 2\times .75}{.25},$$

or

$$W=1200 \text{ lbs.}$$

For general proportions of crane hooks the student is referred to handbooks on Machine Design.

For a section of the form shown in Fig. 91 it is necessary to calculate the moment of inertia about the g–g axis. This is done by dividing the section into rectangles, triangles, and circular sections, and apply the method explained in Art. 23.

PROB. 172. Find the safe load that can be carried by a hook of the form shown in Fig. 91 if the depth at AB is 2 ins., the width at point A is $\frac{7}{16}$ in. and the radius at B is $\frac{7}{16}$ in.

PROB. 173. Find the unit tensile stress in the section CD, if the diameter is $1\frac{1}{8}$ ins.

PROB. 174. Find the unit shearing stress in the section EF, if the depth of the section is $1\frac{9}{16}$ ins., width at narrow end is $\frac{7}{16}$ in. and radius of circular section at E is $\frac{7}{16}$ in.

Art. 54. Machine Frames

The stresses set up in machine frames are usually impossible to analyze mathematically, owing to the peculiar nature of the external load. For example, the frame of a locomotive can be properly designed only from empirical formulæ based on actual experience. Again, the frame of a machine must be rigid enough so as not to distort any of the work being done by the machine, which may mean the use of very high

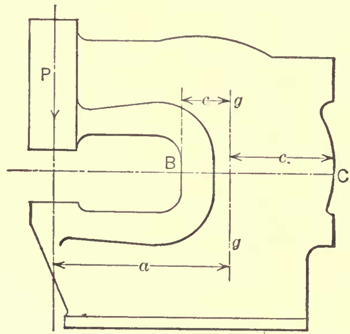


Fig. 92.

factors of safety. In such designs experience and good judgment are often more essential than a mere analysis of the forces acting. There are cases where a somewhat clear analysis of the various stresses can be made to good advantage.

Fig. 92 shows an open type of frame commonly used in the construction of punch presses and riveters. These frames are generally made of cast iron and are built very heavy. The dangerous section of the frame is at BC. The load P will cause a uniform tension on the section BC. Let this tension be represented by S. In addition there will be a bending action on the section BC due to the load P acting at a distance of a units from the neutral axis g–g. Let this bending moment be represented by $P \times a$. At the point B there

will be a unit tensile stress due to the bending moment. Let S equal this stress. Also at the point C there will be a compressive stress due to the bending moment $P \times a$. This compressive stress will be partially counterbalanced by the direct tensile stress due to the load P. Since cast iron is weaker in tension than in compression it is advisable to so design the section that the neutral axis g–g will be located near the point B, thus putting the greater area of the section in tension. Let S equal the safe unit working stress in tension at the point B. It is evident that S will equal the sum of S_t, the direct tensile stress, and S_b, the indirect tensile stress due to bending, hence

$$S = S_t + S_b,$$

but $S_t = \frac{P}{A}$ and $S_b = \frac{Mc}{I}$, where M equals the bending moment $P \times a$, hence $S_b = \frac{Pac}{I}$.

Then

$$S = S_t + S_b = \frac{P}{A} + \frac{Pac}{I},$$

or

$$S = \frac{P}{A}\left(1 + \frac{ac}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

In Equation (101) c is the distance from the neutral axis g–g to the point B, and I is the moment of inertia about the axis g–g. The value of r^2 is obtained from the equation $r^2 = \frac{I}{A}$, where A is the area of the cross-section in square inches. Figs. 46 and 51 show various forms of sections used in the design of punch presses.

As an example of the above, consider the case of a punch press of the form shown in Fig. 92 and let the section BC be of the form and dimensions given in Fig. 46. The 16-in.

width of the section will be placed at B. For this section $c=12.65$ and $Ig=30407$. If the distance $a=30$ ins. let it be required to find the maximum pressure P that can be exerted by the press, so that the unit tensile stress at the point B shall not exceed 3000 lbs. per sq. in. Solving Equation (101) for P gives

$$P=\frac{AS}{1+\frac{ac}{r^2}},$$

where A = the area of section = 122 sq. ins.,

$$c=12.65,$$

and

$$r^2=\frac{I}{A}=250;$$

hence

$$P=145{,}300.$$

Let d equal the diameter of the largest hole to be punched in the plate, and let t equal the thickness of the plate and let S equal the ultimate shearing strength of the material, then the total resistance to be overcome by the punch is $\pi d\times t\times S$; but this resistance can be no greater than the force P, hence

$$P=\pi dtS_s. \qquad \qquad (102)$$

From Equation (102) the maximum diameter of hole may be figured for a given thickness of plate.

Another illustration of combined bending and compression is in the case of a simple overhanging crane of the form shown in Fig. 93. The unit compressive stress at the section mn is equal to the total load P, divided by the area at the section mn. In addition to this unit compressive stress there is a bending stress due to the bending moment exerted by the

force P. This bending moment is equal to the force P times the swing of the crane, or equals $P \times a$. In this case the maximum stress occurs at the point B and is equal to the direct unit compressive plus the indirect compressive stress due to the bending action. Let S_c equal the direct unit compressive stress equal $\frac{P}{A}$ and let S_b equal the indirect

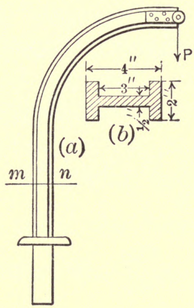


FIG. 93.

compressive stress due to the bending moment, then $S_b = \frac{Mc}{I}$ where $M = P \times a$, therefore

$$S = S_c + S_b = \frac{P}{A} + \frac{P \times a \times c}{I},$$

or

$$S = \frac{P}{A}\left(1 + \frac{ac}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

For example, let the section at mn have the form of an I section. Let the width at point n equal 7 ins., at the point m equal 5 ins., let the thickness of the web be 2 ins. and of the flanges 3 ins. Let the moment arm a equal 24 ins., and depth of section equal 10 ins. If the unit compressive stress at n is not to exceed 10,000 lbs. per sq. in., let it be required to find the maximum load P that can be supported by the

crane. The first step in such a problem is to locate the gravity axis of the section and then determine the moment of inertia of the section relative to the neutral axis g–g. The gravity axis is located by taking moments about the narrow base of the section. Thus

$$5\times3\times1.5+4\times2\times5+7\times3\times8.5=44x$$

or

$$x=5.5 \text{ ins. (approx.)}$$

The moment of inertia about the g–g axis equals

$$\frac{7\times4.5^3}{3}-\frac{5\times1.5^3}{3}+\frac{5\times5.5^3}{3}-\frac{3\times2.5^3}{3}=883.$$

Also

$$r^2=\frac{I}{A}=\frac{883}{44}=20 \text{ (approx.)};$$

hence substituting these values in Equation (103) gives,

$$10{,}000=\frac{P}{44}\left(1+\frac{24\times4.5}{20}\right),$$

or

$$P=70{,}000 \text{ lbs.}$$

Another common illustration of combined bending and tension is in the case of the frame of a side-crank steam engine. The proper frame section can be determined by an analysis of the stresses similar to that in the case of the punch-press frame. Many applications of this principle are to be found in structural work.

Prob. 175. A punch press having a gap of 30 ins. is of the form and dimension shown in Fig. 51. Find the maximum size hole that can be punched in a steel plate $1\frac{1}{4}$ ins. thick, so that the maximum tensile stress on the dangerous section shall not exceed 2800 lbs. per sq. in.

Art. 55. Modulus of Resilience

In Fig. 17, which shows the relation between the load and deformation in a tension test it will be noted as the load is applied the specimen elongates, and hence external work is done upon this specimen. If the load is removed before the elastic limit is reached, the specimen will return to its original length. This external work which is required to stress the bar is called "resilience."

Within the elastic limit the deformation increases in a direct ratio to the load and the curve from no load to the elastic limit becomes a straight line. If the load at the elastic limit is P pounds, then the average resistance offered by the material while the load P is being gradually applied is $\frac{P}{2}$ pounds. Let d equal the deformation at the elastic limit. It is apparent that the work done on the specimen equals the average load $\frac{P}{2}$ times the deformation d, or the *resilience* equals $\frac{Pd}{2}$.

The resilience may be represented by the area included between the y axis and that part of the curve which extends from the origin to the elastic limit. This area takes the form of a triangle whose altitude is the total load at the elastic limit and whose base is the total deformation at the elastic limit. The area of this triangle equals one-half the base times the altitude, or equals $\frac{Pd}{2}$, which is the same as the expression for the resilience.

Elastic resilience is the work done in stressing a bar to the elastic limit.

Ultimate resilience is the work done in rupturing a bar.

Resilience is the property of a material to withstand external work being done upon it. The total elastic resilience

depends upon the cross-sectional area and length of a given specimen. In order to compare the resilience of various materials it is customary to state the resilience in terms of the work done upon a cubic inch of the specimen, in such a case the work done upon a cubic inch of the material is called the *modulus of resilience.* For example, consider the case of a bar of steel subjected to a tensile strain. Let A equal the original cross-section of the bar in square inches, L equal the length in inches, d equal the total elongation at the elastic limit, P equal the load at the elastic limit, E equal the modulus of elasticity of the material, and S equal the unit stress at the elastic limit. Then by definition

$$E=\frac{\text{unit stress}}{\text{unit deformation}},$$

and the elastic resilience equals $\frac{P\times d}{2}$. The unit stress at the elastic limit equals $S=\frac{P}{A}$, or $P=AS$. Likewise the unit deformation equals $\frac{d}{L}$ and hence

$$E=\frac{\text{unit stress}}{\text{unit deformation}}=\frac{\frac{P}{A}}{\frac{d}{L}}=\frac{PL}{dA},$$

or

$$d=\frac{PL}{AE}.$$

Let K equal the modulus of resilience, then substituting these values of P and d in the expression for elastic resilience gives, elastic resilience equals

$$\frac{Pd}{2}=\frac{AS}{2}\times\frac{PL}{AE}=S\times\frac{PL}{2E}.$$

But the modulus of resilience equals the total resistance divided by the volume of the specimen which is AL, hence

$$K=\frac{S\times\frac{PL}{E}}{2AL}=\frac{S^2}{2E}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (104)$$

That is, the modulus of resilience of any material is equal to the square of the unit stress at the elastic limit divided by the modulus of elasticity. For example, the elastic limit of steel is 35,000 lbs. per sq. in., and its modulus of elasticity is 30,000,000, hence the modulus of resilience for steel equals $\frac{35000^2}{30000000}$, or $K=41$. This modulus can be figured for compression in the same manner. Equation (104) is applicable only to either a tension or a compression test, and is never to be used in connection with bending. The modulus of resilience is expressed in inch-pounds since the load is given in pounds and the deformation in inches. For timber in tension $K=3$ in.-lbs.; for cast iron $K=1$ in.-lb., and for wrought iron $K=12$ in.-lbs. When the modulus of resilience is known the work required to stress a specimen of any size can be figured; for example, let it be required to stretch a steel eye-bolt which is 2×4-ins. in cross-section and 6 ft. long, until the unit stress is 20,000 lbs. per sq. in. Assume that the load is to be applied in thirty seconds, find the horse-power required. The total load at the point of maximum stress is $20{,}000\times2\times4=160{,}000$ lbs. The unit deformation due to the unit stress of 20,000 lbs. is $\frac{S}{E}=\frac{20000}{30000000}=\frac{1}{1500}$ ins. The total deformation is $72\times\frac{1}{1500}=\frac{72}{1500}$ ins., since the bar is 72 ins. long. The average load exerted is $\frac{160000}{2}$ $=80{,}000$ lbs., hence the total work done equals $80{,}000\times\frac{72}{1500}$

$=3840$ in.-lbs. The work done per minute is $3840\times 2=7680$ in.-lbs. Hence the horse-power required to stress the bar is $\frac{7689}{12\times 33000}=.02$ (approx.).

PROB. 176. Figure the elastic resilience of a wrought-iron tie rod which is 2×3.5 ins. in cross-section and 10 ft. long when subjected to a unit stress of 15,000 lbs. per sq. in.

PROB. 177. How could the ultimate resilience be figured in the case of a steel specimen under tension?

ART. 56. FORMULÆ FOR ELASTIC RESILIENCE

In the case of beams the elastic resilience is the product of the deflection and the total load applied provided the load is such as to produce a unit stress in the beam not exceeding the elastic limit. If P is the load and d the deflection, then the resilience or work done on the specimen is $\frac{Pd}{2}$, as the load varies from O to P. The modulus of resilience is equal to the elastic resilience divided by the volume of the beam, which is $A\times L$, where A is the cross-sectional area and L is the length of the span in inches. Let K equal the modulus of resilience, then,

$$K=\frac{Pd}{2AL}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

which is a general expression for the elastic resilience of beams.

Several special cases will be considered.

CASE I. Simple beam of span L carrying a concentrated load of P pounds at the center of the span. In this case the deflection equals $\frac{1}{48}\frac{PL^3}{EI}$. The unit stress due to the load P is $S=\frac{Mc}{I}$, but for a concentrated load $M=\frac{PL}{4}$; hence

$S=\frac{PLc}{4I}$, or $P=\frac{4SI}{Lc}$. Substituting this value of P in the expression for deflection gives

$$d=\frac{1}{48}\frac{PL^3}{EI}=\frac{1}{48}\frac{4SI}{Lc}\times\frac{L^3}{EI}=\frac{SL^3}{12Ec}.$$

To express the modulus of resilience in terms of the unit stress substitute the values of P and d in the Equation, $K-\frac{Pd}{2AL}$, which gives

$$K=\frac{4SI}{Lc}\times\frac{SL^2}{12Ec}\times\frac{1}{2AL}=\frac{S^2r^2}{6Ec^2}. \quad . \quad . \quad . \quad (106)$$

In Equation (106) I has been replaced by its equivalent value $I=Ar^2$.

If the total resilience is desired, it may be found by calculating the modulus of resilience from Equation (106) and multiplying this value by the volume of the specimen.

Case II. Simple beam of span L carrying a uniformly distributed load of W pounds per linear foot. In this case the modulus of resilience is found from Equation (105) by substituting for P the total uniform load W and expressing W in terms of the unit stress. For a simple beam uniformly loaded $M=\frac{WL}{8}$, and $S=\frac{Mc}{I}=\frac{WLc}{8I}$, or $W=\frac{8SI}{Lc}$. Likewise $d=\frac{5}{384}\frac{WL^2}{EI}$, hence,

$$K=\frac{Pd}{2AL}=\frac{8SI}{Lc}\times\frac{5WL^3}{384EI}\times\frac{1}{2AL},$$

or

$$K=\frac{5S^2r^2}{12Ec^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

The ultimate resilience of a beam cannot be calculated from any formula, owing to the variable nature of the

relations between load and deformation. Furthermore, the ultimate resilience has little practical value in the design of structures and machines.

PROB. 178. Compute the modulus of resilience of a simple yellow-pine beam of 12 ft. span carrying a concentrated load, such that the unit fiber stress is equal to the elastic limit of the material. The beam is 6×9 in cross-section.

PROB. 179. Compute the modulus of resilience of a light 10-in. steel I beam of 24-ft. span. Beam is uniformly loaded, so that unit fiber stress equals the elastic limit of steel.

ART. 57. STRESSES DUE TO TEMPERATURE

When a steel or iron member is subjected to a change of temperature there is a corresponding decrease or increase in the length of the member, depending upon whether the change in temperature is a decrease or an increase. If the member is free to move there will be no strain put upon the member, but if the member is fixed so that it cannot shorten or lengthen, then a unit stress will be set up which is the same as an equivalent unit stress required to produce the same change of length as that due to the change of temperature. If there is an appreciable change of temperature and the length of the structure is large, then provision must be made by suitable expansion joints to allow for the change of length. If the structure is rigid, the stress due to temperature may be great enough to cause "buckling" or collapse of the structure.

The change in length depends upon the material and upon the change of temperature. The "coefficient of linear expansion" of a given material is the elongation per unit of length per degree difference of temperature. Table XVIII gives the coefficient of expansion for various metals based on a change of temperature of 1° F:

TABLE XVIII

Material.	Coefficient of Expansion.
Steel { Hard	.0000074
Steel { Structural	.0000061
Cast iron	.0000063
Wrought iron	.0000068
Stone and brick	.0000050
Copper	.0000094
Concrete	.0000055

Let C equal the coefficient of expansion, t equal the change of temperature in degrees Fahrenheit, and E equal the modulus of elasticity. The unit stress due to a given elongation is E multiplied by the elongation, as the modulus of elasticity is equal to the unit stress divided by the unit elongation. But the unit elongation for a change of t degrees is Ct, hence the unit stress equals

$$S = CtE. \qquad (108)$$

It will be noted that this unit stress is independent of the length of the specimen. If the specimen is free to change in length, the total elongation equals the unit elongation times the length of the specimen. For example, assume the case of a steel I beam 40 ft. long, which is free to move at the supports. Let the change of temperature be an increase of 70° F., what will be the increase in length of the beam? From Table XVIII, 1° F. will produce a change in length of .0000061 per unit of length, that is, if the beam is 1 ft. in length, for each degree increase of temperature the beam will increase .0000061 ft. in length; hence, for a beam 40 ft. long and a change of temperature of 70° F. the increase in the length of the beam is $.0000061 \times 40 \times 70 = .0171$ ft. $= 2.05$ ins. If the beam decreases in temperature, then the beam will be shortened by an amount equal to the above.

As another example, consider the case of a steel beam which is supported between two rigid columns. Assume the length of beam to be 50 ft. under normal temperature. If the temperature drops 50° F., a tensile stress will be set up in beam, while, if the temperature increases the beam will be in compression. The value of this unit stress is given by Equation (108), thus

$$S = CtE = .0000061 \times 50 \times 30{,}000{,}000$$

$$= 9150 \text{ lbs. per sq. in.}$$

In designing structures proper allowance must be made for the probable change in temperature, for example, if steel rails were laid tight up against one another, when a change of temperature occurred the internal stress would be sufficient to cause a deformation of the rails.

Another common illustration of stress due to change of temperature is in the case of " shrink " fits. For example, the steel tires on the driving wheels of a locomotive are held in place by shrink fits, that is, the tire is turned to an inside diameter somewhat less than the outside diameter of the wheel. The tire is then heated and, of course, increases in diameter. After the proper diameter is secured the tire is slipped over the wheel and allowed to cool, thus contracting and firmly gripping the wheel. The initial diameter must be such that when the tire cools the unit stress due to the contraction shall not exceed a predetermined amount. If the diameter of the wheel is large, and the thickness of the tire very small compared to the wheel, then the diameter of the wheel remains practically constant. In such a case let D equal the diameter of the wheel and d equal the inside diameter to which the tire is turned. Then the unit change in the length of the tire when placed on the wheel is $\frac{(D-d)}{d}$, or this quantity represents the unit deformation. The

corresponding unit stress is $S = E$ times the unit deformation equals

$$E \times \frac{(D-d)}{d}. \qquad \ldots \ldots \quad (109)$$

If the stress is sufficient to cause a decrease in the diameter of the wheel, then the above formula is not applicable. This case is beyond the scope of this text. Large guns are constructed by shrinking a series of jackets onto the barrel of the gun. As an example, consider the case of a cast-iron driving wheel which is turned to a diameter $\frac{1}{1500}$ greater than the diameter of the tire. Here the unit deformation is $\frac{D-d}{d} = \frac{1}{1500}$ and the unit stress is $S = E \times \frac{1}{1500} = 20{,}000$ lbs. per sq. in.

PROB. 180. A wrought-iron tie rod holds together the two heads of a boiler. Find the unit stress in the tie rod when the temperature of the rod is changed 400° F. Assume that the heads of the boiler remain rigid.

PROB. 181. What will be the effect of a change in temperature on a beam which is made up of part concrete and part steel?

ART. 58. REVIEW PROBLEMS

PROB. 182. In a W. I. crank hook of the form shown in Fig. 91 the distance AB is 5 ins., width at the point A is 1 in. and the radius at B is 1 in. The distance p is 8 ins. Find the maximum load that can be placed at W so that the greatest unit stress in tension shall not exceed 7000 lbs. per sq. in.

PROB. 183. In the above hook determine the diameter at CD (see Fig. 91), allowing a factor of safety of six.

PROB. 184. In the above problem select dimensions for the section EF, so that the unit shear shall not exceed 6000 lbs. per sq. in.

PROB. 185. In a punch press of the form shown in Fig. 92 and of the cross-section shown in Fig. 51, determine the distance a if an

inch hole is to be punched in a plate $\frac{7}{8}$ in. thick, assuming the ultimate shearing strength of the plate as 52,000 lbs. per sq. in.

PROB. 186. Figure the elastic resilience when a boiler plate specimen 18 ins. long and $\frac{3}{4} \times 3$ ins. in section is subjected to a unit tensile stress of 18,000 lbs. per sq. in.

PROB. 187. Derive an expression for the elastic resilience of a cantilever beam with a uniformly distributed load.

PROB. 188. Assuming the same cross-section and span compare the relative resilience of the four common types of beams.

PROB. 189. Steel rails are laid in place at a normal temperature of 50° F. What must be the clearance between the ends of the rails to allow an increase of temperature to 110° F.? Assume the rails to be 30 ft. in length.

PROB. 190. A wrought-iron steam line is 500 ft. in length. If the pipe is set in place at 70° F., what will be the increase in the length of the pipe when the temperature is raised to 400° F.

CHAPTER XI

REINFORCED CONCRETE

Art. 59. Cement and Concrete

Concrete, which is one of the most widely used materials of construction, is an artificial stone, made by mixing, in various proportions, Portland cement, sand, and crushed stone, gravel, or cinders. Since it is really stone, it has its greatest value in resisting compressive stresses.

Portland cement, which is the building factor in concrete, received the name Portland because one of the earliest English makers of this product noticed that when it hardened it very closely resembled the stone found in the quarries on the island of Portland, off the south coast of England. It is not a trade name, and should not be confused as such. In this country to-day we have a great many concerns all making Portland cement, but each company uses its own name, as for example, Atlas, Alpha, Lehigh, Edison, Universal and so on. It is made by grinding very finely the clinker which results from burning at a high temperature a mixture of two finely ground stones, one containing a high percentage of lime and the other a high percentage of alumina and silica. These are the three principal elements in Portland cement, and it is necessary to use two different kinds of stone in order to obtain the proper amounts of each. It is this scientific mixing of the powdered stones which really makes Portland cement different from other cements.

Portland cement is mixed with sand to make mortar, and with sand and crushed stone to make concrete. The sand and stone are frequently referred to as aggregates, fine and coarse. Sand is the name given to the material which will pass through a quarter-inch screen, and the remaining material which is too large to pass through the quarter screen is called stone or gravel. The principal requisite of these two materials is that they be hard, durable, clean, and free from loam or other vegetable matter. It is not necessary that the particles of the aggregates be pointed and rough, although this is sometimes helpful. The size of the largest piece of stone to be used in making concrete depends on the purpose for which the concrete is made. They usually are of a size that will pass through a $\frac{3}{4}$-in. or 1-in. diameter ring for reinforced slabs and beams, and may be of 2 ins. or $2\frac{1}{2}$ ins. size for large masses such as engine foundations, columns, footings, retaining walls, etc. In any event, the pieces should be cubical in shape rather than flat or elongated; and should be graded in size from the largest down to the quarter-inch pieces.

The cement sand and stone are mixed together in various proportions by volume. It is customary to express these proportions numerically thus: 1 : 2 : 4, 1 : 3 : 6, which means one part by volume of cement, two parts sand, and four parts stone. The word " part " may mean cubic foot, cubic yard, or even cubic inch. Since a bag of cement holds approximately 1 cu. ft. it is convenient to use the cubic foot as a basis of measurement. The following table will be found helpful in estimating the quantities of materials required to make a given amount of mortar or concrete:

TABLE XIX

Mix.	Quantities of Materials in 1 Cu. Ft. of Mortar.		
	Cement, Bag.	Sand, Cu. Ft.	
1 : $1\frac{1}{2}$	.592	.840	
1 : 2	.496	.919	
1 : $2\frac{1}{2}$	.421	.999	
1 : 3	.367	1.053	
	Quantities of Materials in 1 Cu. Ft. of Concrete.		
	Cement, Bag.	Sand, Cu. Ft.	Stone or Gravel, Cu. Ft.
1 : 2 : 4	.232	.440	.880
1 : $2\frac{1}{2}$: 5	.192	.459	.921
1 : 3 : 6	.164	.470	.940

It is customary to use the 1 : 2 : 4 mix for reinforced concrete and wherever a dense, smooth surface or water-tight concrete is desired. If compressive strength alone is all that is required, a 1 : $2\frac{1}{2}$: 5 mix will probably do and for large masses such as engine beds, foundations, etc., not subject to vibration or shock, a 1 : 3 : 6 can be used. To illustrate how Table XIX is used, suppose we wanted to make 50 cu. ft. of mortar of 1 : $2\frac{1}{2}$ mix, and 125 cu. ft. of concrete of 1 : $2\frac{1}{2}$: 5 mix. We would need for the mortar:

$50 \times .421 = 20.15$, say 20 bags of cement;
$50 \times .999 = 49.45$, say 50 cu. ft. of sand;

Note.—50 cu. ft. of sand equals 1.9, say 2 cu. yds.;

and for the concrete

$125 \times .192 = 240$ bags of cement;
$125 \times .459 = 57.4$, say 58 cu. ft. of sand;
$125 \times .921 = 115.1$, say 115 cu. ft. of stone;

NOTE.—57.4 cu. ft. = 2.12, say $2\frac{1}{8}$ cu. yds of sand;
and 115 cu. ft. = 4.26, say $4\frac{1}{4}$ cu. yds of stone

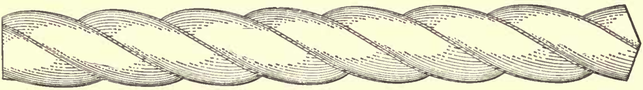
PROB. 191. How many bags of cement and how many cubic feet of sand are required to make the mortar 1 : 3 mix, for a brick pier 8 ft. high by 32 ins. square, if there are 340 cu. ins. of mortar in each cubic foot of brickwork.

PROB. 192. How many barrels of cement, cubic feet of sand and cubic feet of stone will be required to build an engine foundation 10 ft. long by 6 ft. wide and 4 ft. high of stone concrete 1 : $2\frac{1}{2}$: 5 mix. (NOTE.—There are four bags of cement in one barrel.)

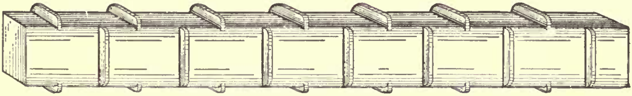
ART. 60. TYPE OF BARS

Concrete as a material used for building construction has strength only in compression. However, the fact that concrete, when hardening, takes a firm grip on any steel rods which may be embedded in it enables us to use concrete, when so reinforced by steel rods, in places where it is subjected to tensile as well as compressive stresses. The steel rods take up the tension and transfer it to the concrete. In this way we can successfully build floor slabs, beams, and columns of reinforced concrete. It is possible to reinforce concrete with plain, round, or square bars, and buildings which have been so reinforced have withstood all stresses and vibrations. It is, however, a fact that some type of "deformed" bar will have a better bond with the concrete and require a greater force to pull it out. For this reason most engineers prefer to use the deformed bars since it is possible to use a smaller number of them, in other words, a square inch of steel in a deformed bar will stand a larger pull than in a plain bar. There are a great many types of deformed bars now being used, and it is impossible to illustrate them all here. A few of the more commonly used forms are shown in Fig. 94 at *a*, *b*, *c*, and *d*. The bar shown at *a* is

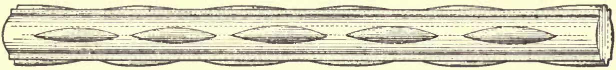
probably the most commonly used of all the deformed bars. There is no patent on this bar, and the additional cost of twist-



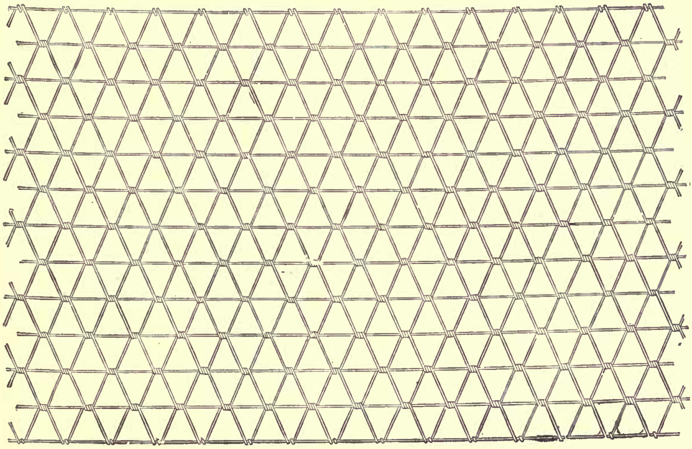
a. Twisted bar.



b. Corrugated square bar.



c. Havemeyer bar.



d. Triangle Mesh Reinforcement.

FIG. 94.

ing is not very great. The bars shown at *b* and *c* are made in both square and round sections, and at *d* is shown one of the common types of reinforcement used in floor slabs.

Art. 61. Slabs and Beams

Reinforced concrete construction can be divided into three general classes: Floor slabs, beams, and columns. Floor slabs are generally thin (about 4 ins. being an average thickness), and their span is usually limited to from 6 to 8 ft., although larger spans may be used. The slab may be supported by steel beams, as shown in Fig. 95, or may be cast

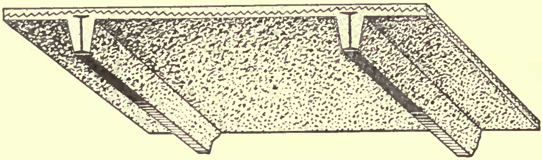


Fig. 95.

as part of the whole floor system, as indicated in Fig. 96. Fig. 95 shows the usual arrangement of concrete and steel in a slab. Notice that the reinforcement (which is here indicated as mesh, but the arrangement would be the same for rods) is not straight through the slab, but is near the lower surface or near the middle of the span and near the upper surface

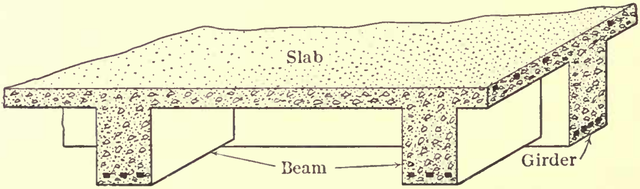


Fig. 96.

where it passes over the beams. Care must be taken to see that this arrangement is maintained. The beams are the members which support the slabs and are, in turn, supported by the girders, which carry the load to the columns or walls. The beams and girders are reinforced by placing the rods

near the lower surface, but since the stresses for which the rods are provided decrease toward the ends of the beams, it is possible to bend up some of the rods diagonally across the beams and in this way help to provide reinforcement against shearing stresses. The way in which the rods are bent is shown in Fig. 97. When the beams or girders form part of

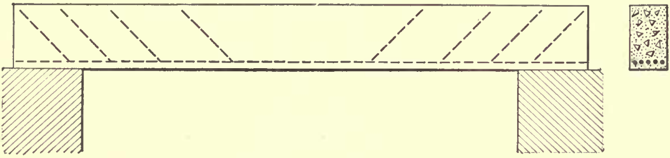
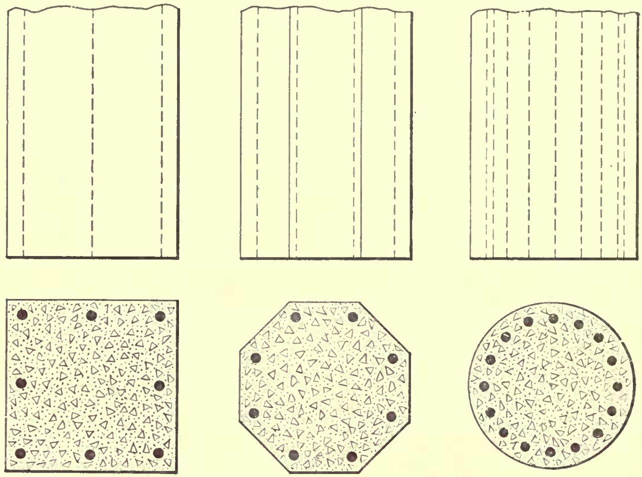


FIG. 97.

a floor system it is customary to bend the roads again and carry them over into the adjoining beam or girder, thus causing the beams to act as continuous beams.

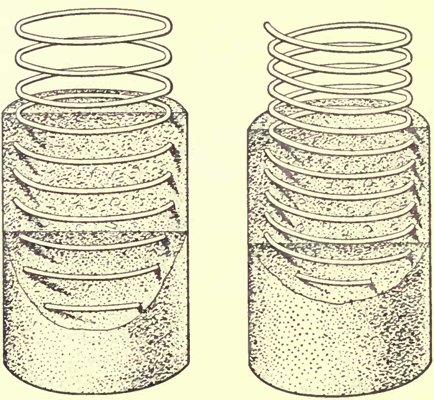
ART. 62. COLUMNS

Concrete columns can be built plain, that is, without any reinforcement, but if they are reinforced they can be made of smaller sections, since the effect of the reinforcement is to increase the allowable unit stress in the concrete. Columns may be reinforced in a number of ways. Fig. 98 shows three column sections, each of which is reinforced with vertical rods only. At *a* is shown a square column with eight reinforcing rods, at *b* an octagonal column with eight rods, and at *c* a circular column with sixteen rods. Fig. 99 shows two columns, the first with "hoop" reinforcing (the hoops must be placed as the column is built), and the second with "spiral" reinforcing. Either of these two methods is effective, but not frequently used on account of difficulties in construction. The most effective and frequently used method of reinforcing columns is shown in Fig. 100. Here we have a combination of the two methods,



(a) (b) (c)

FIG. 98.



"Hoop" "Spiral"

FIG. 99.

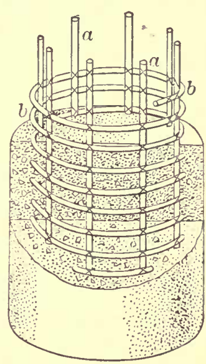


FIG. 100.

and the vertical rods are combined with hoops or spirals. This reinforcement can be used for square, octagonal, or circular sections, and any desired number of rods *a* can be placed inside the spiral *b*. The rods and spiral should be securely fastened together at frequent intervals, so that the spacing shall be maintained during the pouring of the concrete.

Art. 63. Design of Slabs and Beams

Since concrete has very little strength in tension, it becomes necessary to place steel rods in the lower part of concrete beams to take care of the tensile stresses which exist there. It should be kept in mind that this is made possible because, as has been previously stated, the concrete takes a firm grip on the steel when it sets or hardens. The stresses which exist in a reinforced concrete beam are the same as those in any other beam and, therefore, will not be discussed here. There are a number of terms and formulæ which are peculiar to reinforced concrete design and these will be stated and discussed briefly, before illustrating their use by means of numerical examples. The following notation will be used in the formulæ:

Sc = Maximum unit stress in the concrete caused by a given load;

Ss = Unit stress in the steel caused by the same load;

C = Total compression in the concrete;

T = Total tension in the steel;

Es = Modulus of elasticity of steel;

Ec = Modulus of elasticity of concrete;

n = Ratio $\frac{Es}{Ec}$;

d = Distance from top of concrete to axis of steel (not total depth of beam);

b = Breadth of rectangular beam;

As = Area of steel;

p = Steel ration $= \frac{As}{bd}$ or $As = pbd$;

M = Maximum bending moment (in inch-pounds) due to given load.

Both k and j are coefficients which may be considered as follows: If the distance from the top of the beam to the

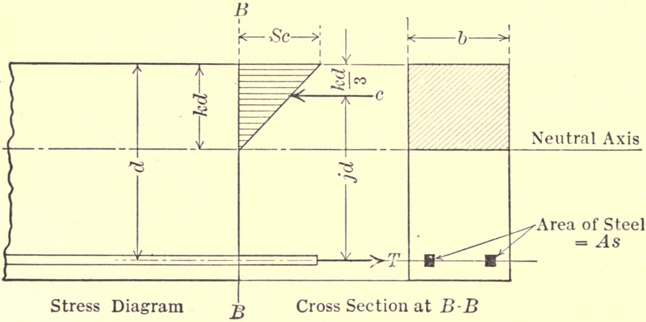


FIG. 101.

neutral axis were called x then the value of k is such that $k = \frac{x}{d}$ or $x = kd$, and it is so shown in Fig. 101. In a similar manner the distance from the center of compression to the steel axis is represented by jd.

Many of the formulæ in reinforced concrete work seem long and complicated, but it is not as difficult to use them as it seems.

The neutral axis is first located by the formula

$$k = \sqrt{2pn + (pn)^2} - pn. \quad . \quad . \quad . \quad (110)$$

Then j is found by remembering that the center of compression is at the center of gravity of the triangle representing the stress in the concrete.

$$jd = d - \frac{kd}{3} \quad \text{or} \quad j = 1 - \frac{k}{3}. \quad . \quad . \quad . \quad (111)$$

The resisting moment of the beam can be found by taking moments about the center of compression or the steel axis instead of about the neutral axis, as was done in the case of ordinary beams. This is due to the fact that all the tension passes through the steel instead of being distributed over the lower part of the section of the beam. Since the resisting moment is equal to the bending moment we can say either

$$M = C \times jd,$$

or

$$= T \times jd.$$

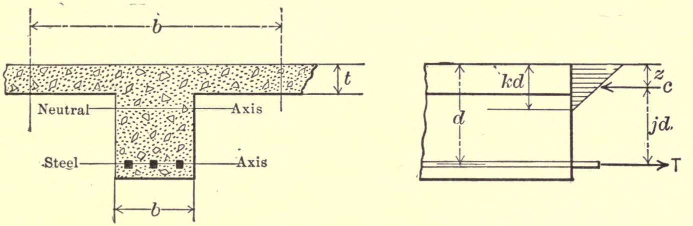


FIG. 102.

From Fig. 102 we see that C, the total compression in the concrete, is equal to $kd \times \frac{Sc}{2} \times b$ and T the total tension in the steel is equal to $A_s \times S_s$. Therefore we have

$$M = \frac{bkdS_c}{2} \times jd, \quad \ldots \ldots \quad (112)$$

or

$$= A_s S_s \times jd. \quad \ldots \ldots \quad (113)$$

From Equation (112) we obtain

$$S_c = \frac{2M}{bkjd^2}, \quad \ldots \ldots \quad (114)$$

and from Equation (113)

$$S_s = \frac{M}{A_s jd} = \frac{M}{pbjd^2}. \quad \ldots \ldots \quad (115)$$

Since $A_s = pbd$.

A simple beam of reinforced concrete could not be designed by finding the value of M due to the load and span, and then assuming values for p, n, and d. k would be found by Formula (110) and j, by Formula (111). Then, by assuming a value for Ss the proper value of b would be found by Formula (115), for which

$$b = \frac{M}{S_s pjd^2}.$$

Finally S_c would be found by Formula (114). This value of S_c must be within the safe limits of concrete in compression or else the whole calculation must be done over again taking a new value for p. It is not usual practice to make as many assumptions as would be indicated by the foregoing, but this has been stated to indicate what the procedure would be like and also to call attention to the fact that the various values of p, n, and S_s are not absolutely fixed, but may vary for different conditions.

Since floor slabs are the most usual form of rectangular beam in general construction, one will be used here for discussion. A beam is most economical when it is so designed that the maximum *allowable* unit stress in the steel is realized under the same load that causes the maximum *allowable* unit stress in the concrete. From this it can be seen that there is a certain amount of steel which will combine with a given area of concrete to cause a given unit stress in the steel to exist at the same time that a given unit stress exists in the concrete. This amount of steel can be indicated by p and it can be found by the formula

$$p = \frac{1}{2} \times \frac{1}{\frac{S_s}{S_c}\left(\frac{S_s}{17 S_c} + 1\right)}. \quad \ldots \ldots \quad (116)$$

Since the value of p depends only on S_s, S_c, and n, it is possible to find values of p, k, and j which can be used for all rectangular beams. Thus, it is recognized as good prac-

tice to use 16,000 for S_s, 650 for S_c, and 15 for n ($E_s = 30{,}000{,}000$ and $E_c = 2{,}000{,}000$).

Solving Formula (116) for p we get

$$p = \frac{1}{2} \times \frac{1}{\frac{16000}{650}\left(\frac{16000}{15 \times 650}\right) + 1} = .00769.$$

From Formula (110) we get

$$k = \sqrt{2 \times .00769 \times 15 + (.00769 \times 15)^2} - .00769 \times 15 = .379$$

and from Formula (111) we get

$$j = 1 - \frac{.379}{3} = 1 - .126 = .874.$$

From Formulæ (114) and (115) we can get

$$M = \frac{S_c bkjd^2}{2},$$

or

$$S_s pbjd^2$$

and by substituting in these the values of p, k, and j just found, we find that

$$M = 107.4bd^2.$$

It is customary to bend the alternate rods in the slab up into the upper part at the ends near the beams and to continue them over the beam near the top surface. For this reason it is the usual practice to consider the value of M as $\frac{WL}{10}$ where W = total load and L = the span in inches.

A portion of the slab 1 ft. wide is considered as a beam and the depth and area of steel figured for this. Thus, suppose we had a floor carrying a live load of 150 per sq. ft. supported on beams 6 ft. 6 in. apart. The total load on a strip 12 ins. wide would be $6\frac{1}{2} \times 150 = 975$ plus the weight of

the slab itself. This will probably be about $3\frac{1}{2}$ ins. thick, which with the top floor will cause a dead load of say 45 lbs. per sq. ft. $6\frac{1}{2}\times 45=292.5$ lbs., say 300 lbs. W then $=1275$ lbs.

From this $M=\dfrac{WL}{10}=\dfrac{1275\times 6.5\times 12}{10}=9945$ in.-lbs.

Since $b=12$ and $M=107.4bd^2$, we can say

$$d^2=\frac{9945}{107.4\times 12}=7.7,$$

from which $d=2.78$.

This value of d is known as the effective depth and we must add about $\frac{3}{4}$-in. of concrete below the steel which will make the slab $3\frac{1}{2}$ ins. thick as assumed.

The amount of steel to use in the slab is found by

$$\begin{aligned} A_s &= pbd \\ &= .0077\times 12\times 2.78 \\ &= .265. \end{aligned}$$

This means that we must use .265 sq. in. of steel in each foot of width of slab, and this can be made up by using a mesh or rods. If a mesh reinforcement is used the area, size, etc., can be obtained from the manufacturers' catalog. If rods are used the spacing is figured so that the correct amount of steel shall be placed in each foot of width. It is customary to use rods from $\frac{1}{4}$-in. to $\frac{3}{4}$-in. in size and vary the spacing to suit the condition. For example, for the above slab if we decide to use $\frac{3}{8}$-in. square bars, the area of each is .14 sq. in. and we will need $\dfrac{.265}{.14}=1.89$ rods in each 12 ins. of slab. This would mean that the rods would be placed $\dfrac{12}{1.89}=6.35$ say $6\frac{3}{8}$ ins. apart. Rods in slabs should not be spaced more than $2\frac{1}{2}$ times the slab thickness apart nor nearer together

than $2\frac{1}{2}$ times the diameter of rod. If $6\frac{3}{8}$ ins. is considered as too wide spacing for the rods we can use $\frac{5}{16}$-in. rods which have an area of .098. The spacing can be found directly by

$$\frac{12\times .098}{.265}=4.43, \text{ say } 4\tfrac{1}{2}\text{-in. ctrs.}$$

Rectangular beams are designed in just the same way as slabs with the exception that they are now always 12 ins. wide (they should not be less than 6 ins. wide), and the steel area cannot always be made up as near to the correct value as with slabs. The procedure is to choose a number of rods which will give an area as near as possible to that required and then investigate for S_s and S_c using the formulæ

$$S_s=\frac{M}{A_sjd} \quad \text{and} \quad S_c=2pS_s,$$

remembering that

$$p=\frac{A_s}{bd}.$$

Tee Beams. In a floor system of all concrete construction, it is customary to cast the beams and slabs at one operation. When this is done the slab can be considered as part of the beam and figured as part of the compressive area. Fig. 102 shows a diagram of a tee beam with the slab forming the flange of the beam. Notice that the width b is measured on the flange and the width of the stem is called b^1, the slab thickness is t, and the compressive stress is considered to act at a distance z below the top of the flange.

The width b is limited and is usually taken as about 12 or 14 times t provided this is not more than the $\frac{\text{span of the beam}}{4}$ or more than the distance between beams. The width b^1 is usually about $2\times t$ and must be wide enough to permit the proper placing of the steel rods, which should not be spaced

nearer together than $2\frac{1}{2}$ diameter. They do not, however, have to be all in one row, but may be in two or more layers. The value of d can be assumed or can be approximated by the formula

$$d=\frac{V+75b^1t}{150b^1};$$

where

$$V = \text{Maximum shear}$$
$$= \text{End reaction.}$$

Then an approximate value of A_s can be obtained by the formula

$$A_s=\frac{M}{\left(d-\frac{t}{2}\right)S_s}. \qquad \qquad (117)$$

This will usually give an area of steel a little larger than that actually required, and for this reason some engineers consider the beams to be designed at this point. It is not always safe to assume that the stresses will all be within proper limits and, therefore, the following procedure should be carried out. It should be noted that this method neglects the compressior which exists in the stem of the beam below the slab.

Having found A_s by Formula (117) we can find kd by the formula

$$kd=\frac{2ndA_s+bt^2}{2nA_s+2bt}. \qquad \qquad (118)$$

If this gives a value of kd which is less than t, then the beam is not considered to be a tee beam, but it is figured as a rectangular beam. If, however, the value of kd is equal to more than t, we proceed as follows: Find Z by the formula

$$Z=\frac{3kd-2t}{2kd-t}\times\frac{t}{3}. \qquad \qquad (119)$$

and jd, by the formula

$$jd=d-z. \qquad \qquad (120)$$

Then S_s can be found by

$$S_s=\frac{M}{A_s jd},$$

and S_c by the formula

$$S_c=\frac{Mkd}{bt\left(kd-\frac{t}{2}\right)jd}.$$

If the values of S_s and S_c are not safe, new values must be taken for b and d and the calculations made again.

The design of a tee beam is best illustrated by an example, as follows: Suppose the beams supporting the slab previously designed were 18 ft. long. Find their proper size and amount of steel reinforcing.

The total load supported by the beam will be $6.5\times18\times(150+45)$ plus the weight of the stem of the beam. The stem will probably be about 8 ins. wide and 10 ins. deep below slab. It will weigh $8\times10=80$ lbs. per lin. ft. or $18\times80=1440$ lbs. This gives a total load of 22800 lbs. + 1440 lbs. = 24240 lbs. and we will use $M=\frac{WL}{10}$ again, as we will bend part of the rods up at the ends of the beam and carry them over into the next beam. M, therefore, is $\frac{24240\times18\times12}{10}=523,580$ in.-lbs.

Figuring for a value of d we get

$$d=\frac{V+75b^1t}{150b^1}=\frac{12120+75\times8\times3.5}{150\times8}=11.85, \text{ say } 12 \text{ ins.}$$

Checking our assumed value of 10 ins. for depth of stem we get, 12 ins. + $1\frac{1}{2}$ ins. (fireproofing below rods which is not considered as part of the beam) = $13\frac{1}{2}$ ins. − $3\frac{1}{2}$ ins. (thickness of slab) = 10 ins.

Next figuring for A_s by Formula (117)

$$A_s=\frac{M}{\left(d-\frac{t}{2}\right)S_s}=\frac{523580}{\left(12-\frac{3-5}{2}\right)\times 16000}=3.19,$$

say, three 1-in. sq. rods.

$$kd=\frac{2ndA_s+bt^2}{2nA_s+2bt}=\frac{2\times 15\times 12\times 3+42\times 3.5\times 3.5}{2\times 15\times 3+2\times 42\times 3.5}=4.27 \text{ ins.}$$

(NOTE.—The value of b was used as $12\times t$).

This value of k means that we have a true tee beam since the neutral axis is below the flange.

Then finding Z by Formula (119)

$$Z=\frac{3kd-2t}{2kd-t}\times\frac{t}{3}=\frac{3\times 4.27-2\times 3.5}{2\times 4.72-3.5}=1.15 \text{ ins.}$$

$$jd=d-z=12-1.15=10.85;$$

$$S_s=\frac{M}{A_sjd}=\frac{523580}{3\times 10.85}=16085,$$

which is near enough 16,000 to pass, and

$$S_c=\frac{Mkd}{bt\left(kd-\frac{t}{2}\right)jd}=\frac{523580\times 4.27}{42\times 3.5(4.27-1.75)10.85}=556;$$

which is well below 650, the allowable value of S_c. If the value of S_c found by the formula is considered too small we can assume either a smaller value of b or smaller value of d and refigure the beam, also if the value of S_c found by the formula is too large we must take a larger value of d or b or both and try again.

Stirrups. Practically every beam has a tendency to fail as shown in Fig. 103, and to prevent this the beam should be provided with shear rods or stirrups, these are placed in the beam as indicated in Fig. 103 and the spacing

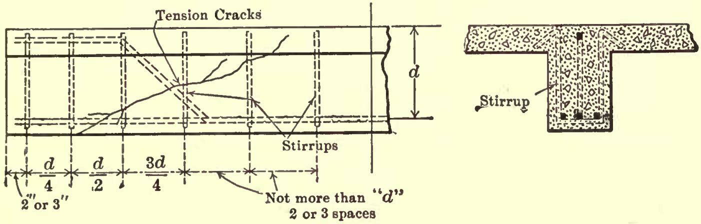


FIG. 103.

there is for a uniform load. The stirrups are usually made of $\frac{1}{4}$-in. to $\frac{1}{2}$-in. square bars.

ART. 64. COLUMNS

The usual method of reinforcing concrete columns is shown in Fig. 100. There are various formulæ by which the safe load on a reinforced concrete column can be figured but they are seldom used. The reason is that the principal object of the steel is to increase the strength of the concrete, rather than to support much of the load itself. This increased value of the compressive strength of the concrete varies with the amount of steel which is used, and some of the rules governing this will be stated and illustrated. One rule is that if from 1 to 4 per cent of vertical reinforcement together with about 1 per cent of spirals or hoops are used, the unit stress in the concrete can be increased 45 per cent over the strength of plain concrete and the steel can be figured at 7000 per sq. in. of section. The amount of reinforcement used varies with the load on the column, but it is not customary to use more than 4 per cent. To illustrate how the calculations are made, suppose we had a load of 350,000

to support by a round reinforced concrete column 12 ft. long. (The length of a concrete column should not be more than 15 times the diameter of the column.) An approximate area of the column can be found by dividing the load by about 700, assuming that we will use $2\frac{1}{2}$ per cent of vertical reinforcement, and that the value of S_s for plain concrete is 450, the value S_c for the reinforced concrete will be about 650. Therefore 350,000 divided by $700=500$. This would mean a column 24 ins. in diameter. The area of this column is 455 sq. in. $2\frac{1}{2}$ per cent of this is 11.35 sq. in., so we will use eleven 1-in. square rods. The strength of the column is found to be $455\times650+11\times7000=373{,}000$, which is well over the required strength. The amount of spiral necessary is found as follows: If we use 1 per cent we will need 1 per cent of the volume of the column as spiral. The volume of a section of column 1 ft. long is 5428.8 cu.; in. 1 per cent of this is 54.3 cu. in. This means that we must have 54.3 cu. in. of spiral in each foot of length of the column. If we use $\frac{3}{8}$-in. round rod for spiral there will be $75.4\times.11=8.29$ cu. in. in each turn around the column. There will be $\frac{54.3}{8.99}=6.55$ turns around the column per foot of length, or we could say that the pitch of the spiral will be $\frac{12}{6.55}=1.83$, say $1\frac{7}{8}$ in.

It is necessary to have a protective coating of at least $1\frac{1}{2}$ ins. outside the steel which cannot be figured as adding any strength to the column. This will make the column 27 ins. in diameter.

Art. 65. Practical Review Problems

Prob. 193. Design a floor slab, using the critical amount of reinforcement to carry a load of 200 lbs. per sq. ft. on a span of 6 ft. 6 ins. The load includes the weight of the slab. The slab is not part of a floor and must be considered as a simple beam.

PROB. 194. A rectangular reinforced concrete beam supports a load of 2500 lbs. per ft., including its own weight, over a span of 12 ft. Assuming that $b = 10$ ins., $d = 22$ ins. and $A_s = 2.5$ sq. ins., what stress is produced in the concrete and in the steel?

PROB. 195. Design a rectangular reinforced concrete beam to support a load of 1500 lbs. per foot, in addition to its own weight, on a span of 16 ft. Design to the nearest half inch and assume $b = 12$ ins. Find d, total depth and area of steel for main reinforcing.

PROB. 196. Design a slab to support a load of 250 lbs. per sq. ft., in addition to its own weight, on a span of 8 ft. Slab is part of a floor and, therefore, the rods can be bent up and carried over into the next span.

PROB. 197. A tee beam has a flange $3\frac{1}{2}$ ins. thick and supports a load of 1800 lbs. per lin. ft., which includes its own weight, on a span of 15 ft. The value of d is 14 ins. and $b = 40$ ins. Find how much steel is required and the values of the stress in the concrete and the steel. Consider beam as a simple beam.

PROB. 198. Design a tee beam on a span of 20 ft. to support a load of 2200 lbs. per lin. ft., which includes its own weight. The flange is 4 ins. thick, and part of the main reinforcing rods are bent up and carried over into the next beam. Take $b^1 = 12$ ins.

PROB. 199. Design a round reinforced concrete column to support a load of 325,000 lbs. Use 3 per cent of vertical rods and 1 per cent of spirals.

PROB. 200. Design a square reinforced concrete column to support a load of 550,000 lbs., using 4 per cent of vertical steel rods and 1 per cent of spirals.

PROB. 201. Prepare a schedule of spacing for stirrups for beam in Prob. 198.

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